EXPERIMENTAL DETERMINATION OF THE OBLIQUITY OF THE ECLIPTIC

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1. INTRODUCTION

25% of the world's population still think the sun revolves around the earth. In fact, the planets move around the sun. It is also little known that the ecliptic plane is not the same as the celestial Equator, as the Earth is tilted 23.5 degrees with regard to the ecliptic plane.

In this study we deal with this and many more things related to the obliquity of the ecliptic, by only using a gnomon, one of the most important objects in physics, and a tape measure.

2. OBJETIVES

1. To learn about the historical antecedents with respect to the calculation of the obliquity of the ecliptic.

2. To study the mathematical relation between the Sun's highest point, latitude and the Sun's declination.

3. To study, experimentally, the evolution of the Sun's declination during a long period of time.

4. To estimate the obliquity of the ecliptic.

5. To establish, experimentally, the latitude of the place from which we observe the sun (Lleida) from its declination value.

3. BACKGROUND

3.1 Some history

One of the great discoveries of man in history, the importance of which is sometimes underestimated, was the gnomon. Our ancestors observed the shadow of a vertical stick nailed to the floor of a horizontal surface change in length and position from sunrise to sunset, and noticed that that its length was minimal at solar noon.
The next step was to observe the evolution of the length and direction of the shadow of the gnomon for long periods of time. The shadows were shorter in summer and longer in winter, but the direction of the shadow at solar noon was always the same: the one which today is known as the **meridian line,** which coincides with the direction North-South. A simple gnomon provided valuable information on which time of year it was.
If every day the top of the shadow of the gnomon was indicated for one year, changing curves were obtained of which three forms stood out:

a) Straight line: equinoxes

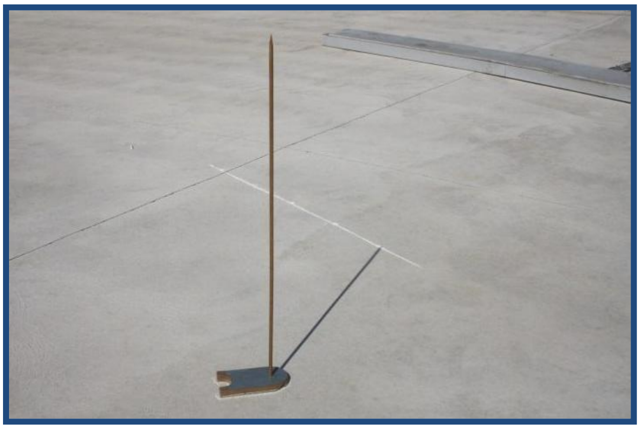


Image 1: Evolution of the shadow of the gnomon during the spring equinox of 2014. Playground INS Guindàvols

b) Convex hyperbole: summer solstice

Image 2: Evolution of the shadow of the gnomon during the summer solstice of 2014. Playground INS Guindàvols.

c) Concave hyperbole: winter solstice

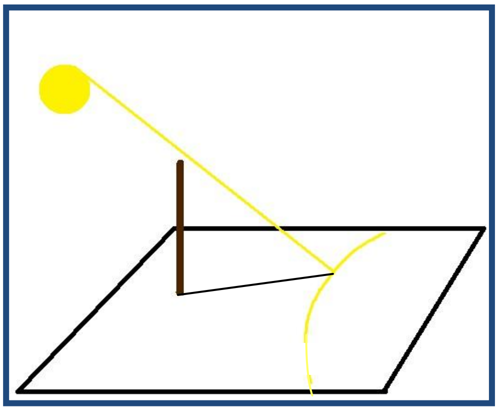


Image 3: Evolution of the shadow of the gnomon during the winter solstice (drawing)

Apparently, the Egyptians circa 3550 BC already knew the form the shadow of an obelisk would take throughout the day and in different seasons. The obelisk served as a sundial and a calendar. (Image 4)

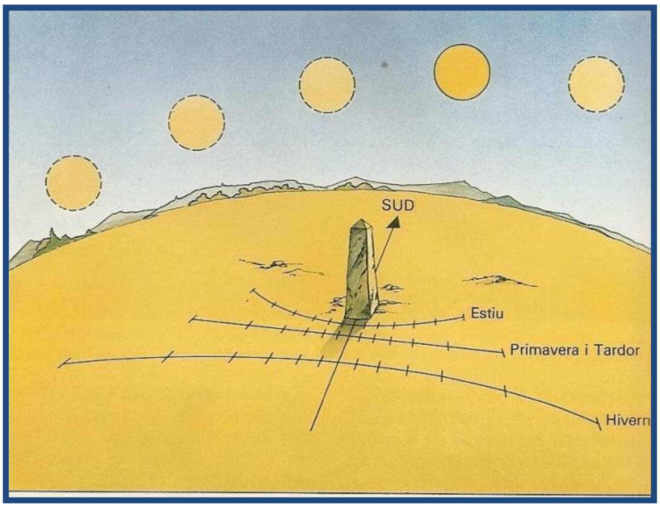


Image 4: Egyptian obelisk. The position of the shadow gave information on the part of the day and the season[2]

Presumably one of the great challenges of science in antiquity was to find out about the origin of the regularity observed in the shadow of the gnomon in different seasons.

According to Ptolemy in the Almagest[1], Aristarchus of Samos explains the origin of the equinoxes and solstices by stating that the axis of rotation of the Earth was inclined to the perpendicular of the plane of its orbit (**obliquity of the ecliptic**). (Image 5).

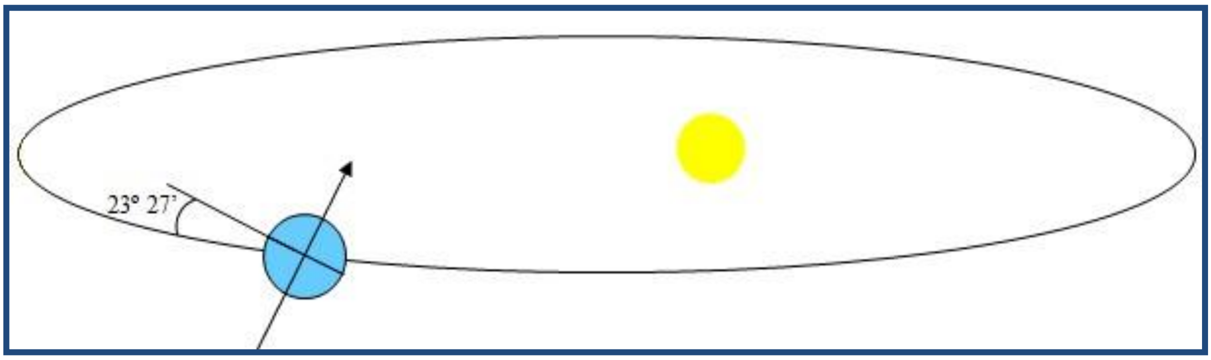


Imagen 5: The plane of the Equator is not the same as the plane of Earth's orbit around the Sun, but it is inclined at an angle of 23 degrees 27'(obliquity of the ecliptic).[3]

Thus he managed to explain, according to the heliocentric model, the change in the seasons. **But how can the angle of inclination be measured?**

It is good to keep in mind the astronomical knowledge about the apparent movement of the sun in ancient Greece in this respect:

3.2 Apparent movement of the Sun

The paths of the Sun on the celestial sphere were known from a geocentric model of the universe. (Image 6). The Sun draws every day an arc of apparent motion in the sky from east to west, passing always through the exact South at solar noon (solar noon, i.e. when the sun is at its highest point in the sky). That arc, Parallel Capricorn, reaches its minimum size on the winter solstice, December 21. On that day the sun rises in the east (ortho) quite close to the South and sets in the west (sunset), also pretty close to the South. As the astronomical year progresses, this arc becomes greater every day, so that every morning the sun rises closer to this exact spot and sets at another one closer to the exact West. It reaches these exact rising and setting points in the spring equinox. On that day we can rightly say that the sun rises in the east and sets in the west. The position of the sun at the equinox coincides with the area that became known as "celestial Equator", that is, in the middle of the sky. But the year progresses and the arc of apparent movement continues to widen. Therefore, from that day on until Cancer Parallel reaches its maximum extent in the summer solstice, the sun rises in the Northeast and sets in the west. From that moment onwards the arc begins to narrow, back to its minimum length at the following winter solstice.

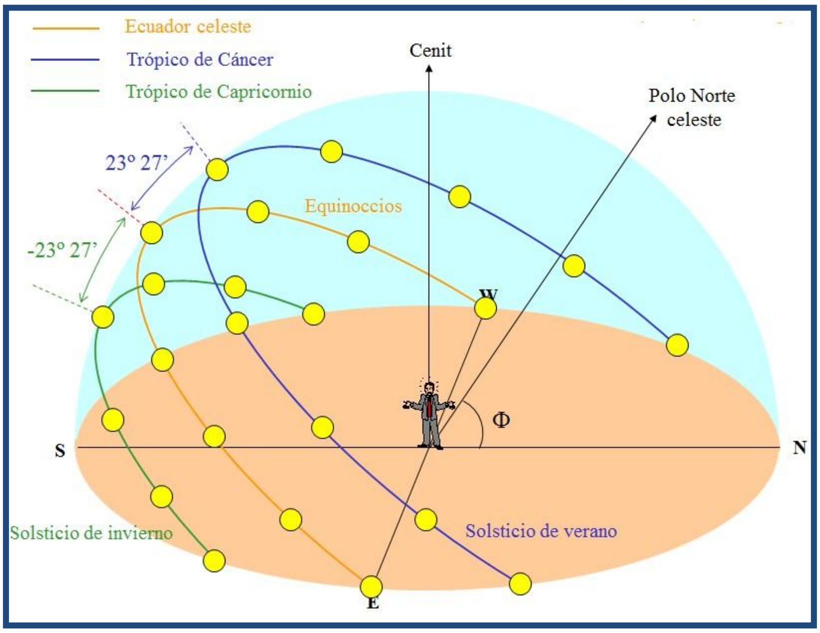


Image 6: Apparent paths of the Sun in the northern sky[3].

Plutarch[4] describes how Thales of Miletus in 558 a. C. made the first estimate of the obliquity of the ecliptic. He found that the arc between two tropics was equivalent to 8/60 of the circumference, or, which is the same, 48º. Therefore the angle of inclination of the earth (obliquity) was 24. (Image 7)

Eratóstenes discovered that half the Meridian Arc between their two tropics was 11/83 of the full circle (which corresponds to 23º, 51´, 18´´).

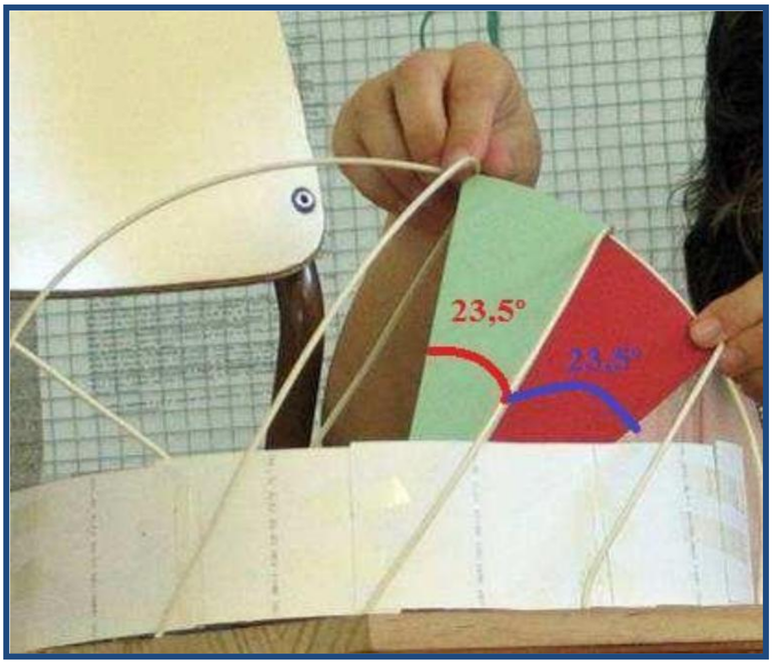


Image7: Model to visualize the apparent trajectory of the sun on the first day of each season[5]. The angle between two trajectories of the starting day of two consecutive seasons is about 23, 5º.

3.3 Relationship between the maximum height of the sun, latitude and the declination of the Sun

The earth does not only rotate on its axis (which causes day and night), but also moves in a slightly eccentric orbit around the Sun, which is its focus. The axis of rotation of the Earth is tilted 23.5 degrees relative to the line perpendicular to the plane of the ecliptic (the plane containing the orbit of the Earth).
The combination of the motion of the Earth, with the fact that its axis is tilted, leads to the seasons. (Image 8)

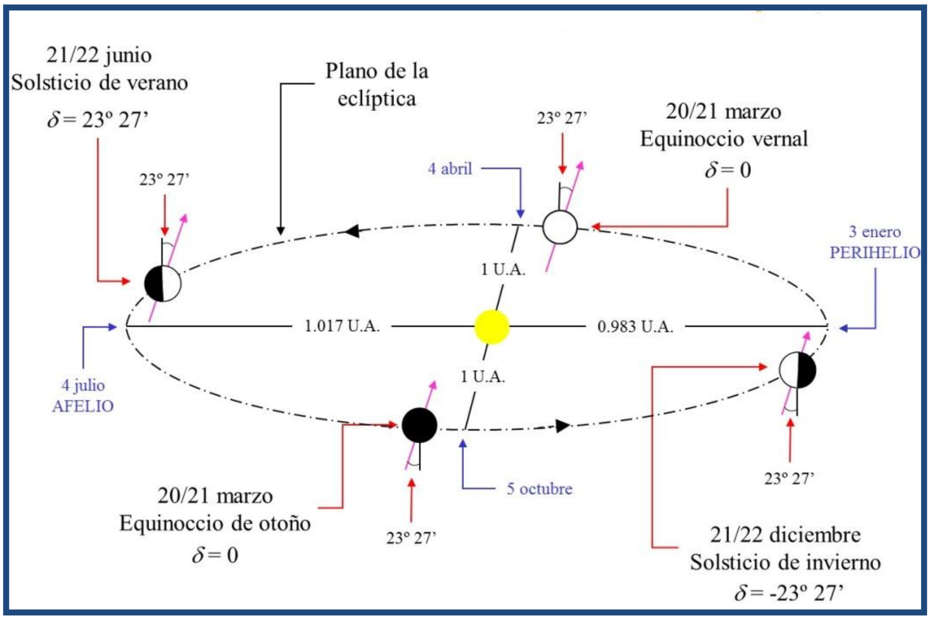


Image 8: Earth orbit. Stations [3]

If we study the apparent movement of the sun over a year from inside the celestial sphere, we can see that it moves against its starry background following an imaginary curved path which is called **Ecliptic**. This path on the celestial sphere is a circle that forms the celestial Ecuator with an angle of 23 ° 27 ' **obliquity of the Ecliptic**. The angle formed by the sun's rays impinging on the Earth and the celestial Equator is called the **angle of solar declination**, δ.
The ecliptic intersects the celestial Equator at two points: the vernal equinox and the autumnal equinox in which the sun's declination is zero degrees. The declination of the Sun changes throughout the year: on the day of the summer solstice the sun's declination is high (23.5), and on the day of the winter solstice, minimal (-23,5º).

Significantly, **the obliquity of the ecliptic coincides with the maximum solar declination** (23.5 °) (Image 9).

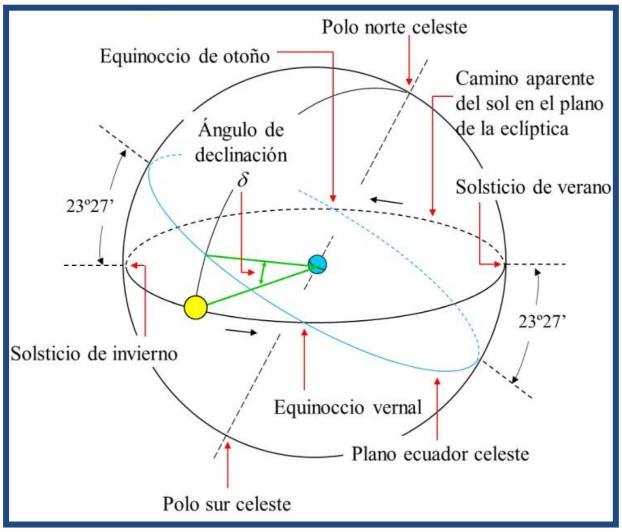


Image 9: Annual movement of the Sun on the celestial sphere. Solar declination angle.[3]

If an observer on the horizon of a geographical location in the northern hemisphere and latitude, Φ, studies the apparent movement of the sun on a spring or summer day, they notice that just after its departure, the movement will ascend and describe a circle parallel to the celestial Equator, reaching its maximum height, α_{max}, at the exact moment that it crosses the local meridian (solar noon). The solar declination, δ , is the angle that verifies:

$$\alpha_{max} - \delta + \Phi = 90$$ **(Equation 1)**

Knowing the latitude of Lleida, Φ = 41,6 °, and calculating the height of the sun at solar noon, one can determine the declination of the sun. If these measurements are made daily over a year we'll know the evolution of the declination of the sun. **The maximum value of the solar declination gives us the obliquity of the ecliptic.**

$$\delta = \alpha_{max} + \Phi - 90$$ **(Equation 2)**

3.4 Solar Noon

Solar noon takes place every day when the sun is at its highest point in the sky. This happens when it crosses the meridian Celeste. (Image 10)
During the Equinoxes, the sun is vertically above the places on the Equator. So, in the Equinoxes, at Solar Noon, the Sun is in the South in the Northern Hemisphere and in the North in the Southern Hemisphere. At the Solstices, the Sun is vertically above the places in the Tropics.

The shadows cast by sunlight are shortest when the Sun is at the highest place in heaven. This moment is called Solar Noon. The length of the shortest shadow enables us to know the angle between the Sun's rays with the vertical line (on each geographic location where the measurement is made). By comparing these angles in two different places, it is possible to estimate the circumference and the radius of the Earth.

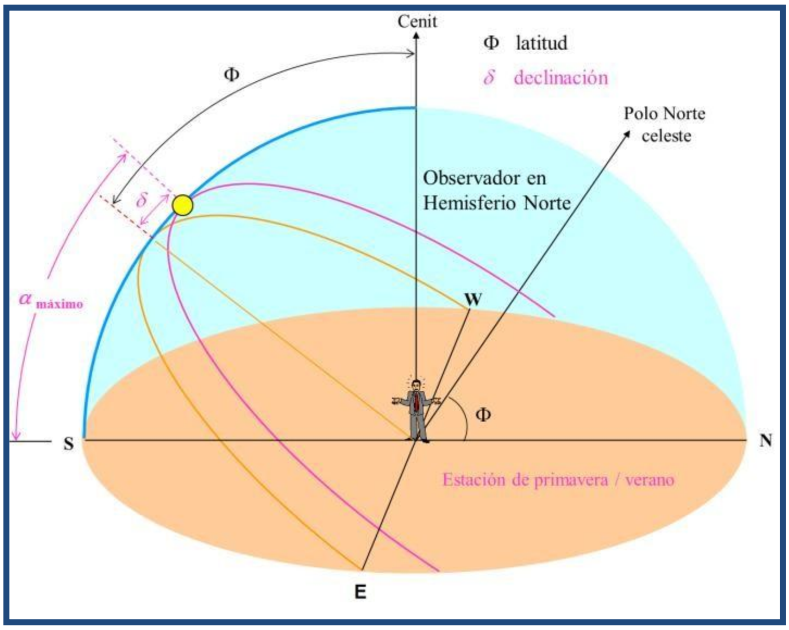


Image 10: Maximum height of the Sun at solar noon[3]

In what ways can we calculate solar noon?

a) We need to know sunrise and sunset of the day on which we want to calculate solar noon. This information can be obtained at the website, tutiempo [6]. The solar noon is the intermediate value between the time of departure and sunset.

b) Consulting the program Stellarium [7]. The solar noon is the moment when the sun crosses the local meridian. (Image 11)

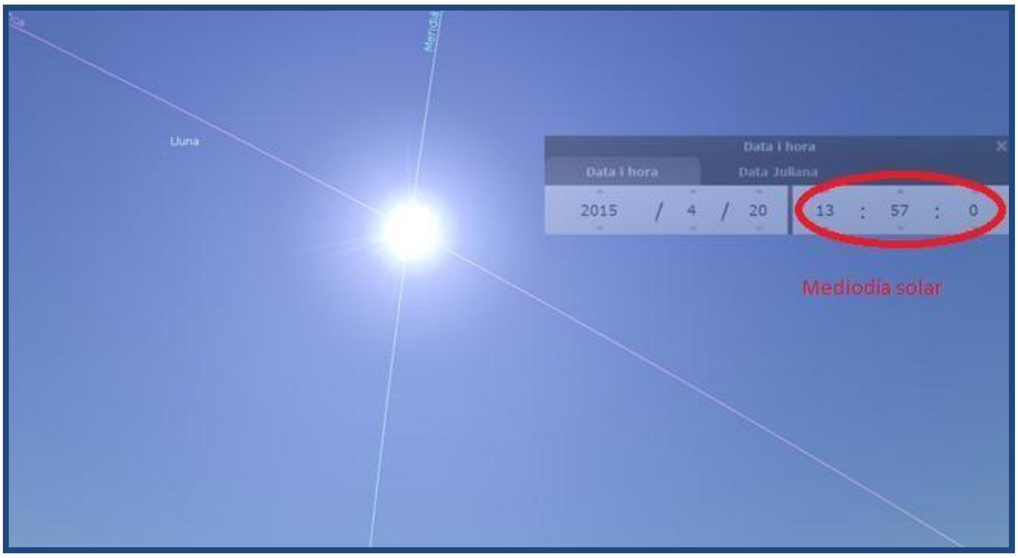


Image 11: Determining solar noon with the Stellarium program

4. EXPERIMENTAL DESIGN

4.1 Experiment 1: determination of the obliquity of the ecliptic

4.1.1Description of the experiment

To measure the obliquity of the ecliptic the evolution of the declination of the Sun will be monitored over a long period of time (six months). To do this, we'll measure each day the Sun's altitude at solar noon with the help of a gnomon. By measuring the length of the gnomon, Lg, and the length of the shadow at solar noon, Ls, we'll calculate the maximum height of the Sun (Image 12).

$$\mathrm{tg}\ \alpha_{\text{máx}} = \frac{L_{gnomon}}{L_{sombra}}$$ **(Equation 3)**

$$\alpha_{\text{máx}} = arctg\left(\frac{L_{gnomon}}{L_{sombra}}\right)$$ **(Equation 4)**

From Equation 2 we can obtain the maximum height of the sun, α_{max}. If we also know the latitude of the place, Φ, we can determine the sun's declination, δ, using Equation 2.

$$\delta = \alpha_{max} + \Phi - 90$$

The maximum value obtained from the declination of the Sun (summer solstice), will coincide with the obliquity of the Ecliptic (angle of inclination of the axis of rotation of the Earth relative to the perpendicular to the plane of the Ecliptic).

4.1.2 Materials

• Gnomon	• Measuring tape
	

4.1.3 Graphic representation

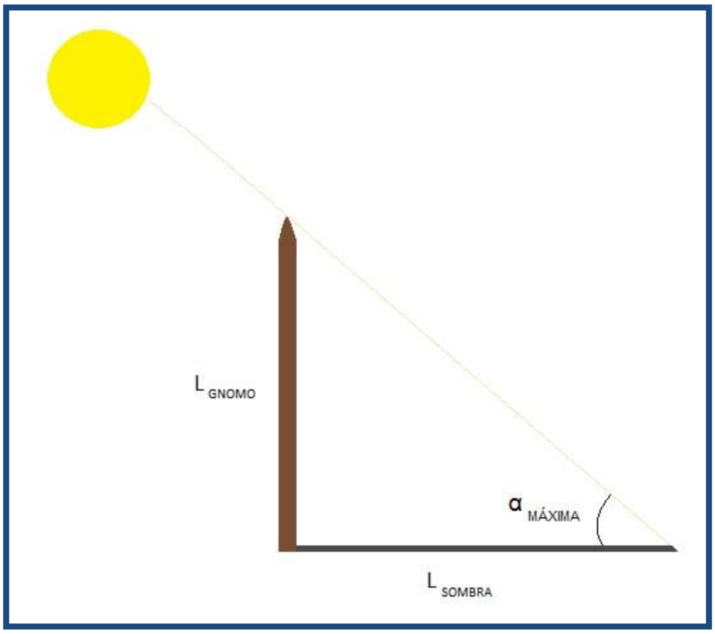


Image 12: Graph representation of the experiment

4.1.4 Obtained results. Analysis and discussion

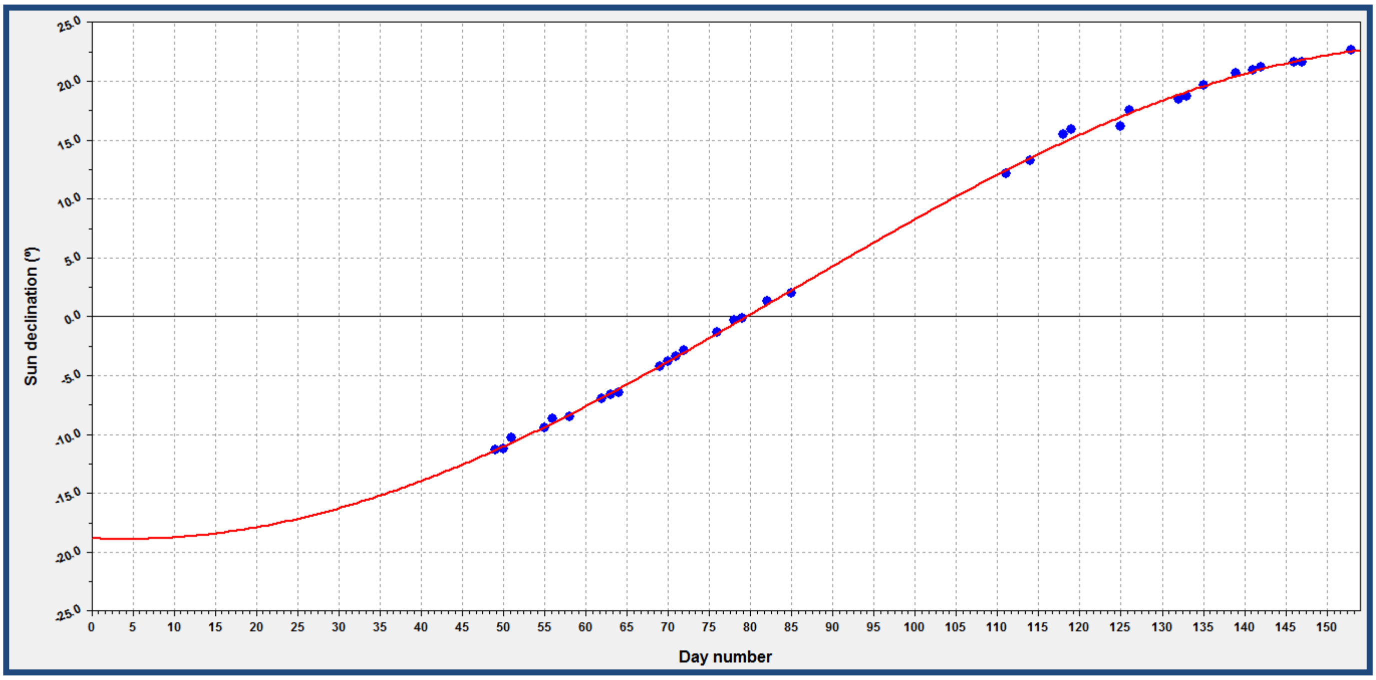
4.1.4.1 Table of the Sun's declination evolution.

DATE	NUMBER DAY	SUNRISE (h:min)	SUNSET (h:min)	SOLAR NOON (h:min)	MAXIMUM HEIGHT OF THE SUN (º)	DECLINATION OF THE SUN (º) $\delta = \alpha_{max} + \Phi - 90$ (ϕ of Lleida = 41,6º)
18/2/15	49	7:52	18:32	13:12	37.10	-11.30
19/2/15	50	7:51	18:33	13:12	37.20	-11.20
20/2/15	51	7:49	18:34	13:11	38.11	-10.29
24/2/15	55	7:44	18:39	13:11	38.99	-9.41
25/2/15	56	7:42	18:40	13:11	39.78	-8.62
27/2/15	58	7:39	18:43	13:11	40.13	-8.50
3/3/15	62	7:33	18:48	13:10	41.70	-6.95
4/3/15	63	7:32	18:49	13:10	42.07	-6.58
5/3/15	64	7:30	18:50	13:10	42.07	-6.45
10/3/15	69	7:22	18:56	13:09	44.17	-4.23
11/3/15	70	7:20	18:57	13:08	44.58	-3.82
12/3/15	71	7:19	18:58	13:08	45.00	-3.40
13/3/15	72	7:17	18:59	13:08	45.57	-2.83
17/3/15	76	7:10	19:04	13:07	47.04	-1.36
19/3/15	78	7:07	19:06	13:06	48.42	-0.29
20/3/15	79	7:05	19:07	13.06	48.42	-0.14
23/3/15	82	7:00	19:10	13:05	49.70	1.30
26/3/15	85	6:55	19:14	13:04	50.86	2.00
21/4/15	111	7:12	20:42	13:57	60.59	12.19
24/4/15	114	7:08	20:45	13:56	61.67	13.27
28/4/15	118	7:02	20:50	13:54	63.89	15.49
29/4/15	119	7:00	20:51	13:55	64.34	15.94

5/5/15	125	6:52	20:57	13:54	64.57	16.17
6/5/15	126	6:51	20:58	13:54	65.96	17.56
12/5/15	132	6:44	21:05	13:54	66.90	18.50
13/5/15	133	6:43	21:06	13:54	67.14	18.74
15/5/15	135	6:41	21:08	13:54	68.10	19.70
19/5/15	139	6:37	21:12	13:54	69.08	20.68
21/5/15	141	6:35	21:14	13:54	69.32	20.92
22/5/15	142	6:34	21:15	13:54	69.57	21.17
26/5/15	146	6:31	21:18	13:54	70.06	21.66
27/5/15	147	6:30	21:19	13:54	70.06	21.66
2/6/15	153	6:26	21:24	13:55	71.06	22.66
3/6/15	154	6:26	21:25	13:55	71.07	22.67
9/6/15	160	6:24	21:29	13:56	72.07	23.67
21/6/15						**Ecliptic obliquity**

4.1.4.2 Graphic representation of the evolution of the Sun's declination throughout one semester.

If we represent the Sun's declination in function of the day of the year with the free program Curve Expert[8], we obtain the following graphic:



It will be noticed that the Sun's declination has been increasing since the first observation day (18/2/15), going from -11,30º to 0,14º (20/3/15, Spring Equinox) until 23,67º (9/6/15). Presumably, the Sun's declination is going to increase until reaching a maximum value at the Summer Solstice (21/6/15), which is going to be the maximum and going to concur with the obliquity of the ecliptic.

Unfortunately, the Sun's declination in the Winter Solstice could not be determined because of climatic reasons, and as the project has to be presented before the Summer Solstice.

4.2 Experiment 2: Determination of latitude

The latitude of an observation place can determined both by day and by night and in various ways[9].

If we want to establish the geographic coordinates of an observation point during the day, we will need know the declination of the sun on the observation day. This can be found in the Astronomical Yearbook or by carrying out the experiment described above. We know the declination of the sun on equinoxes and solstices; on equinox days the Sun moves along the Equator so its declination is zero, while in the summer solstice the Sun reaches its maximum declination (δ = 23º27') and in winter the minimum (δ = -23º27').

On the other hand it is also necessary to know the maximum height that the Sun will reach above the horizon on the day that we carry out the experiment. The Sun will reach this height on the solar midday, which will have to be defined as accurately as possible.

With the help of this equation and all the results obtained in the experiment 1 we can determinate the latitude.

$$h = 90 - \delta + \phi$$ **(Equation 5)**

4.2.1 Obtained results. Analysis and discussion

DATE	NUMBER DAY	SUN DECLINATION (º)	LATITUDE (º)
8/2/15	49	-11.30	41.7
19/2/15	50	-11.20	41.5
20/2/15	51	-10.29	41.6
24/2/15	55	-9.41	41.6
25/2/15	56	-8.62	41.6
27/2/15	58	-8.50	41.37
3/3/15	62	-6.95	41.35
4/3/15	63	-6.58	41.35
5/3/15	64	-6.45	41.48
10/3/15	69	-4.23	41.6
11/3/15	70	-3.82	41.6
12/3/15	71	-3.40	41.6
13/3/15	72	-2.83	41.6
17/3/15	76	-1.36	41.6
19/3/15	78	-0.29	41.29
20/3/15	79	-0.14	41.44
23/3/15	82	1.30	41.6
26/3/15	85	2.00	41.14
21/4/15	111	12.19	41.6
24/4/15	114	13.27	41.6
28/4/15	118	15.49	41.6
29/4/15	119	15.94	41.6
5/5/15	125	16.17	41.6
6/5/15	126	17.56	41.6
...	...	...	...

5. CONCLUSIONS

1. The ecliptic obliquity was determined in ancient times by Aristarchus Samos.
2. One can determine the declination of the sun by measuring its height at solar noon and knowing the latitude of the observation place by using this formula:

$$\delta = \alpha_{max} + \Phi - 90$$

α_{max} = maximum Sun height

δ = Sun declination

ϕ = latitude

3. We have determined the declination of the sun during a long period of time, but not long enough to get to the summer solstice (June 21) because the project is to be handed in before this date. In any case, we will continue measuring the declination of the Sun next year.

4. We have designed a procedure to determine the latitude of an observation place based on the declination of the sun. The average value for Lleida is 41,6º.

6. A WORD OF THANKS

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