



Catch a Star 2015

Title:

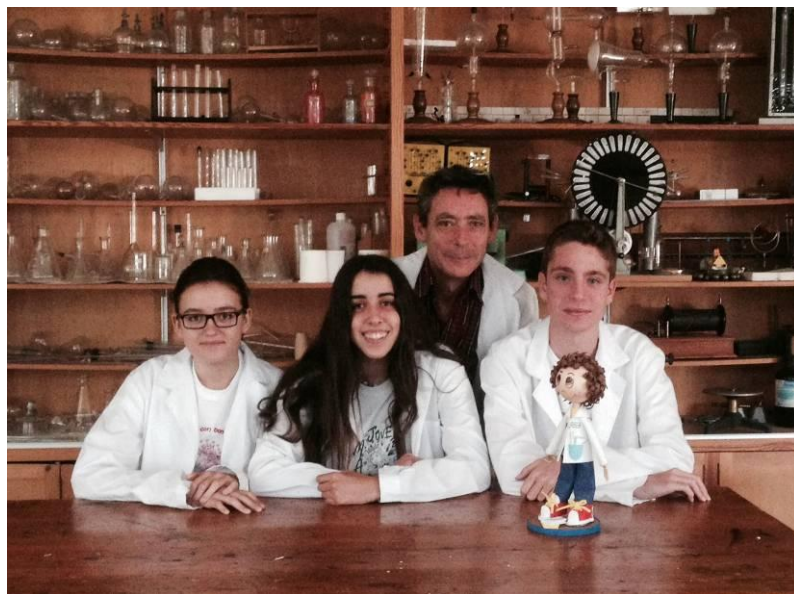
Testing the universal gravitation law to the limit

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Abstract

Our students of 3rd of ESO have checked some equations of Universal Gravitation by combining with typical orbital motion equations. We started calculating - with great accuracy - the masses of almost all celestial bodies in our solar system, those which have satellites in rotation. This study may be considered as a routine exercise for Science students of ESO and Baccaureate and it is sure that these calculations were known for centuries. We know we are not the first to do it. However, we wanted to extend our study - with recent discoveries - to calculate masses of stars with *exoplanets* and finally, based in very recent discoveries too, we achieve estimate the mass of an invisible and mysterious object that reigns in the heart of *Sagittarius A**: a colossal *black hole* with the mass of five million suns. In this respect we are confident that we are a team of pioneering students!

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Development

Introduction

In 1687 Isaac Newton presented his book *Philosophiae Naturalis Principia Mathematica* in which he presented his law of universal gravitation. This law states that two bodies of masses ***M*** and ***m*** always attract with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance ***R*** between them:

$$F = (-)G \frac{Mm}{R^2}$$

Where ***G*** is a universal constant calculated later (1798) by Henry Cavendish. The value of this constant is: $G = 6,67 \cdot 10^{-11} \frac{Nm^2}{Kg^2}$

If we divide by ***m*** on the law of Universal Gravitation, we obtain:

$$\frac{F}{m} = (-)G \frac{M}{R^2}$$

This expression - according to the second law of Newton - represents acceleration. Is the acceleration of gravity that a body of mass m (at a distance R) experiences in the presence of a larger mass M :

$$g = (-)G \frac{M}{R^2}, \text{ (It's always an attractive acceleration with radial direction).}$$

Isaac Newton knew well how to relate this acceleration with the uniform circular motion (circular first approximation) that describes the Moon around the Earth.

This is the starting point of our study.

To describe a circular motion, we need a physical quantity called *angular velocity*. This is the ratio of the rotated angle (measured in radians) by the time it takes to do:

$$\omega = \frac{\varphi}{t}$$

For practical purposes it is a magnitude very easy to calculate: it's calculated by simply dividing a full circle (2π rad) between the time taken for one complete rotation performed (*orbital period*)

$$\omega = \frac{\varphi}{t} = \frac{2\pi}{T_{orbital}}$$

Moreover, all bodies that experience circular movements have an acceleration that informs us of the change of direction of the linear velocity. It is the *centripetal acceleration* whose calculation is immediately:

$$a_c = \omega^2 \cdot R \text{ (Product of the angular velocity squared by the orbital radius)}$$

Newton suspected that the *centripetal acceleration* and *gravity acceleration* might be related. And how? The centripetal acceleration experienced by the Moon on his journey around the Earth is precisely the acceleration of gravity generated by our planet wherever the Moon is:

$$a_c = g \quad \Rightarrow \quad a_c = (-)G \frac{M}{R^2}$$

And then, a main consequence of this equality is that we can easily calculate the mass M of the central object around which revolves the second:

$$M = \frac{a_c R^2}{G}$$

And this is the ultimate goal of the study presented.

Early studies

Immediately we wanted to check this last expression with our own movement around the Sun.

Earth makes a complete orbit in: $365,265 \text{ días}$; time spent immediately to seconds ($31.558.118,4 \text{ s}$) and it will give the angular velocity with which we move around the Sun:

$$\omega = \frac{2\pi}{T_{\text{orbital}}} = \frac{2\pi}{31.558.118,4} = 1,99 \cdot 10^{-7} \text{ s}^{-1}$$

The orbital radius of the Earth (the average distance from Earth to the sun) is: $1,49598 \cdot 10^{11} \text{ m}$

Therefore we will have an angular acceleration:

$$a_c = \omega^2 \cdot R = 0,0593 \text{ m/s}^2$$

And by using this result, we can know the mass of the central object that is drawn along our annual movement:

$$M = \frac{a_c R^2}{G} = 1,9897 \cdot 10^{30} \text{ Kg}$$

It is, of course, the mass of our Sun.

Fast, easy and highly accurate method (the value found in sources of information is: $1,9891 \cdot 10^{30} \text{ Kg}$, observing a failure of a six per ten thousand. This is a very exact value).

Of course, we tried with the rest of the planets in our solar system:

Sol	1,9891E+30 kg					
	P. Orbital	P. Orbital (s)	ω (rad/s)	Radio orbital (m)	a_c (m/s ²)	M Sol (Kg)
Mercurio	87d 23,23h	7600428	8,27E-07	57894376000	0,039566	1,98823E+30
Venus	224,701d	19414166,4	3,24E-07	1,08209E+11	0,011334	1,98969E+30
Tierra	365,256d	31558118,4	1,99E-07	1,49598E+11	0,00593	1,9897E+30
Marte	686,971d	59354294,4	1,06E-07	2,27937E+11	0,002554	1,98963E+30
Júpiter	11a 315d 1,1h	374359262,4	1,68E-08	7,78412E+11	0,000219	1,99198E+30
Saturno	29a 167d 6,7h	929638353,6	6,76E-09	1,43E+12	6,52E-05	1,98896E+30
Urano	84,323 326a	2,66E+09	2,36E-09	2,87E+12	1,6E-05	1,97789E+30
Neptuno	164a 288d 13h	5,20E+09	1,21E-09	4,49825E+12	6,57E-06	1,99197E+30

Figure 1. Mass of the sun from the orbital data of planets

(In any case we obtained higher errors than 1%)

It seems that the connection of the movement of all the planets with the central force that binds us all is clear. And it seems also clear that is verified and with great accuracy, Kepler's third law:

$$M = \frac{a_c R^2}{G} = \frac{\omega^2 R^3}{G} = \frac{4\pi^2 R^3}{T^2 G} \quad \Rightarrow \quad \frac{R^3}{T^2} = \frac{M \cdot G}{4\pi^2} = \text{cte}$$

(Well, basically, we have been applying all the time that fact in all the examples of calculation)

Extending the process a step further

The same procedure carried out in the previous point may be used to calculate the mass of any planet that has stable satellites revolving around it.

1.- We did so with the Earth itself taking the orbital parameters of the Moon, the International Space Station and any of the geostationary satellites orbiting the Earth.

The data used are the same as before: the orbital period of the satellite and the average radius of its orbit and nothing else. The same calculations are performed as above with the difference that this time the body found is that of our own Earth.

Tierra	5,9736E+24 kg						
	P. Orbital	P. Orbital (s)	ω (rad/s)	R. Orbital (m)	a_c (m/s ²)	M Tierra (Kg)	
Luna	27d 7h 43,7m	2360622	2,7E-06	384400000	0,002723	6,033E+24	
	P. Orbital	P. Orbital (s)	ω (rad/s)	R. Orbital (m)	a_c (m/s ²)	M Tierra (Kg)	
ISS	92m 52s	5572	0,00113	6775000	8,614832	5,9284E+24	
	P. Orbital	P. Orbital (s)	ω (rad/s)	R. Orbital (m)	a_c (m/s ²)	M Tierra (Kg)	
Sat geoest	23h 56m 4s	86164	7,3E-05	42164000	0,224207	5,976E+24	

Figure 2. Mass of Earth from orbital data of its satellites

The results are also excellent (no error less than 1%)

2.- With *Phobos* and *Deimos* (using the same procedure) we obtained the mass of **Mars**:


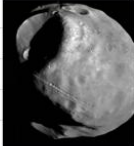
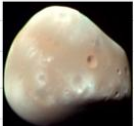
Marte	6,4185E+23 Kg						
	P. Orbital	P. Orbital (s)	ω (rad/s)	R. Orbital (m)	a_c (m/s ²)	M Marte (Kg)	
Phobos	0,31891023 d	27553,8	0,000228	9377200	0,487606978	6,42822E+23	
							
	P. Orbital	P. Orbital (s)	ω (rad/s)	R. Orbital (m)	a_c (m/s ²)	M Marte (Kg)	
Deimos	1,262 d	109036,8	5,762E-05	23460000	0,077900734	6,42794E+23	
							
							La Tierra
							0,1 M _{Tierra}
							(5,9736E+24 kg)

Figure 3. Mass of Mars from the orbital data of its two satellites

And we obtained the mass of the parent star (*51 Pegasi*) with very good approximation and checking that it is a star similar to our sun.

After that, we studied HD 209458b, named "*Osiris*" (1999) in the region of Pegasus too. This is similar to the previous exoplanet and as we can see, orbiting another Sun-like star (HD 209458)

HD 209458	1,13	M _☉				Calculada	
	P. Orbital (d)	P. Orbital (s)	ω (rad/s)	R. Orbital (m)	a_c (m/s ²)	HD 209458 (Kg)	
HD 209458 b	3,5	302400	2,1E-05	7031099922	3,0354245	2,24978E+30	
"Osiris"		302400		0,047 UA		1,131 M _☉	
						Estrella tipo solar (1999)	

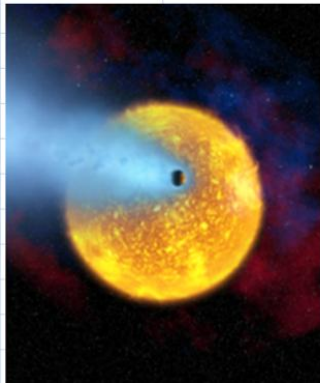


Figure 6. Mass of HD 209458

These two exoplanets have something else in common: they are orbiting so close to its parent star that solar winds make their atmospheres be lost quickly.

Within the system of planets around *Kepler 186*, we find a planet with the size of Earth (*Kepler 186F*) and, moreover, it is placed in the so-called *habitable zone*

Kepler 186	0,48	M _☉				Calculada	
	P. Orbital (d)	P. Orbital (s)	ω (rad/s)	R. Orbital (m)	a_c (m/s ²)	M Kepler 186 (Kg)	
Kepler 186b	3,8867	335810,88	1,9E-05	5654799512	1,9796455	9,49065E+29	
		335810,88		0,0378 UA		0,477 M _☉	
Kepler 186c	7,2673	627894,72	1E-05	8586917778	0,8598514	9,50544E+29	
		627894,72		0,0574 UA		0,478 M _☉	
Kepler 186d	13,343	1152835,2	5,5E-06	12880376666	0,3826078	9,51667E+29	
		1152835,2		0,0861 UA		0,478 M _☉	
Kepler 186e	22,4077	1936025,28	3,2E-06	18191101076	0,1916005	9,5058E+29	
		1936025,28		0,1216 UA		0,478 M _☉	
Kepler 186f	129,946	11227334,4	5,6E-07	58732124033	0,0183943	9,51279E+29	
		11227334,4		0,3926 UA		0,478 M _☉	

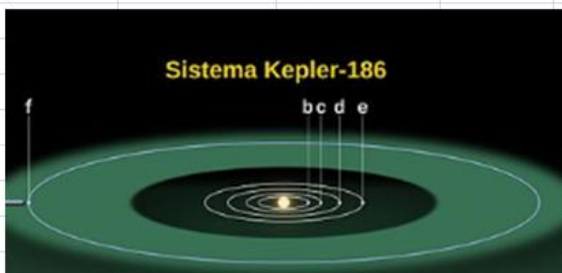


Figure 7. Mass of HD 209458

In this case we find that the mass of the parent star is very small (average value obtained: 0.478 solar masses). It is a *red dwarf star*. However, at a distance of 0.39 AU, it can provide a comfortable temperature.

Finally (in this section) we show the closest star system to ours we have found. It has a solar-type star with a very similar distribution of planets to ours. It is the star 55 Cancri system and its planets:

55 Cancri	0,95	M_{\odot}				Calculada	
	P. Orbital (d)	P. Orbital (s)	ω (rad/s)	R. Orbital (m)	a_c (m/s ²)	55 Cancri (Kg)	
55 Cancrib	14,65162	1265899,968	5E-06	17203755129	0,4238227	1,88064E+30	
		1265899,968		0,115 UA		0,945 M_{\odot}	
55 Cancric	43,93	3795552	1,7E-06	35903488966	0,098389	1,90149E+30	
		3795552		0,24 UA		0,956 M_{\odot}	
55 Cancri d	4517,4	390303360	1,6E-08	7,86436E+11	0,0002038	1,88982E+30	
		390303360		5,257 UA		0,950 M_{\odot}	
55 Cancri e	0,736537	63636,7968	9,9E-05	2333726783	22,750614	1,85766E+30	
		63636,7968		0,0156 UA		0,934 M_{\odot}	
55 Cancri f	260	22464000	2,8E-07	1,16836E+11	0,0091403	1,87064E+30	
		22464000		0,781 UA		0,940 M_{\odot}	

Figure 8. Planetary system and mass of 55 Cancri

Extending the procedure to the limit

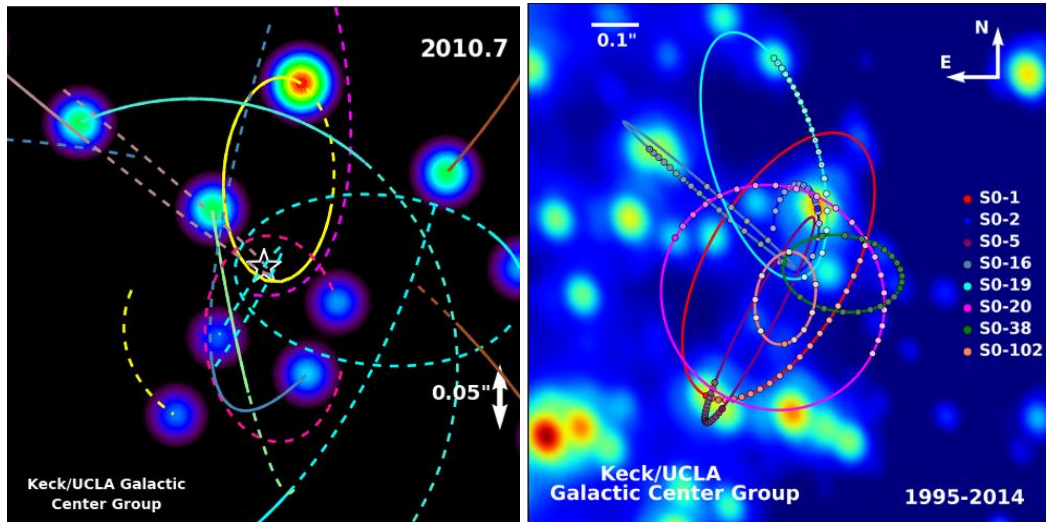
We recently found a sequence of images that showed us how some stars move very quickly around an invisible point (called *Sagittarius A **) that is at the center of our galaxy.

Our sun is located at a distance of 25,900 light years from the center of our galaxy (the own *Sagittarius A **) and, despite the great distance, we dared to calculate its mass.

We got enough data from three of the many stars orbiting *Sagittarius A **.

A sequence of images made by the *Keck/UCLA Galactic Center Group* showed us observation dates in each frame. In this way we were able to estimate - with some approximation, frankly - periods of rotation of these three stars.

In addition, from a new graphic, which gave us a scale of angular observation (also the *Keck/UCLA Galactic Center Group*), we could perform some calculations that helped us to find the means orbital radii.



Figures 9 & 10. Stellar graphics moving around Sagittarius A* (Keck/UCLA Galactic Center Group)

Although the method differs slightly from the previous one (we include here a simple trigonometric calculation to find the orbital radius from the observed angle), we proceeded basically equal.

SO-2 en Sagitario A		25900 años luz	
φ (rad)	6,283185308	escala	0,2"
t_0	1996,4	0,79	2,88
t	2011,2	4,68	6,81
T_{orb} (días)	5405,7	0,203827751	"
ω (rad/s)	1,34528E-08	9,88185E-07	rad
R (m)	1,21069E+14	semieje a	
a_c (m/s ²)	0,021910939		
M (Kg)	4,81505E+36		
M_\odot	2420715,636	$M_\odot=1,9891E+30$	

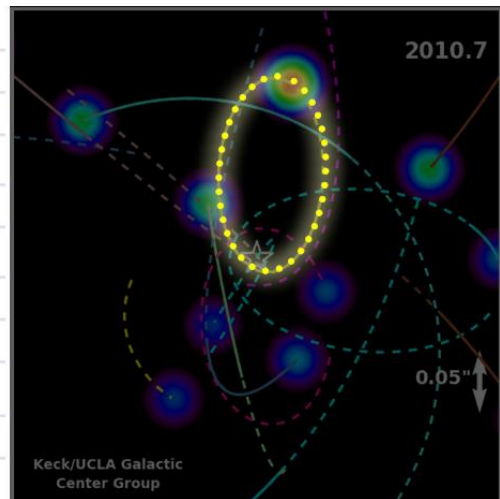


Figure 11. Calculations using data orbital of S0-2

SO-102 en Sagitario A		25900 años luz	
φ (rad)	6,283185308	escala	0,2"
t_0	2000,6	0,79	2,88
t	2011,8	6,35	8,6
T_{orb} (días)	4090,8	0,215311005	"
ω (rad/s)	1,7777E-08	1,04386E-06	rad
R (m)	1,2789E+14	semieje a	
a_c (m/s ²)	0,04041581		
M (Kg)	9,91052E+36		
M_\odot	4982415,075	$M_\odot=1,9891E+30$	

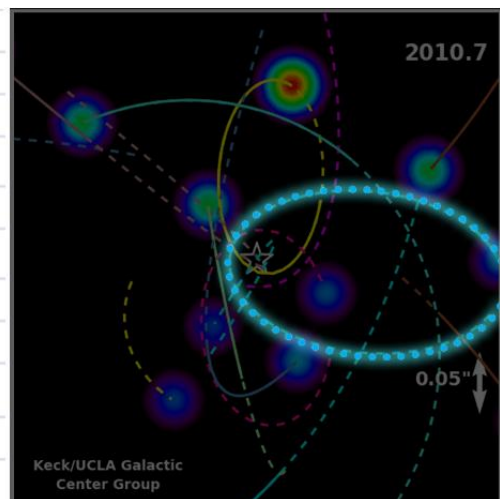


Figure 12. Calculations using data orbital S0-102

S0-38 en Sagitario A		25900 años luz	
φ (rad)	6,283185308	escala	0,2"
t_0	1995,5	0,79	2,88
t	2013,7	6,4	9,3
T_{orb} (días)	6647,55	0,277511962	"
ω (rad/s)	1,09397E-08	1,34542E-06	rad
R (m)	1,64836E+14	semieje a	
a_c (m/s ²)	0,019726954		
M (Kg)	8,03593E+36		
M_\odot	4039983,707	$M_\odot=1,9891E+30$	

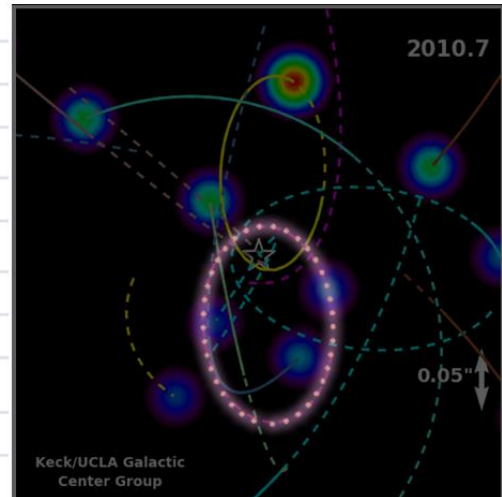


Figure 13. Calculations using data orbital of S0-38

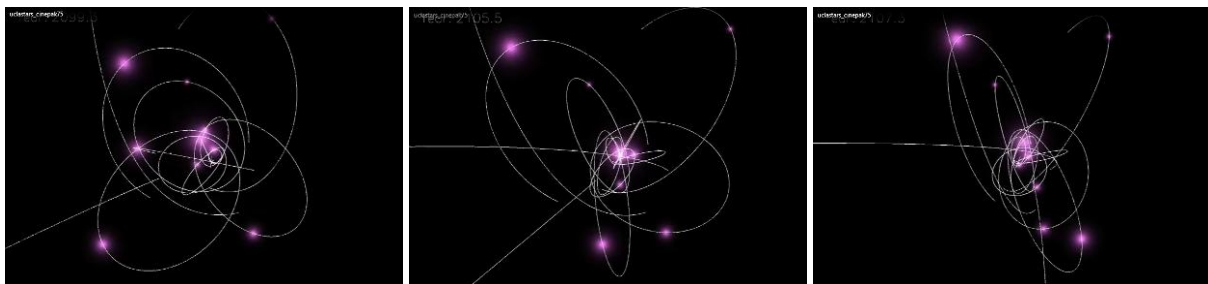
The first thing we noticed was the enormous value from the result of the three cases (several million solar masses). The mysterious central object responsible for these movements - and invisible on any image - might be, without doubt, a huge *black hole*.

This is a truly amazing and exciting result!

And moreover, we wondered why we had obtained three very different results with each other (we see there are differences of over 30 %!). And so far, our results were incredibly accurate!

The answer lies in a new series of graphics by the *Keck/UCLA Galactic Center Group* that shows the perspective of the paths followed by the stars orbiting the galactic center.

The prospect deceives us when we try to measure the actual size of the orbit. If the major axis of the orbit is close to our observation line, we will always get a lower value than the real and consequently will get lower results in our final calculation of the central mass.



Figures 14, 15 & 16. Oriented orbits in the three directions of space around Sagittarius A* (Keck/UCLA Galactic Center Group)

Certainly - we thought - this is the case of our first result found by observing S0-2



Figure 17. Reduced orbit (unreal) offered by the perspective for Sagittarius A* (homemade design)

In contrast, with S0-102 we obtained the highest value, which would indicate that the prospect of its orbit allows us to see it more approach with its real geometry.

The correct procedure in science is to take the *arithmetic mean* of the values found for the most likely value (in our case: 3.814 million solar masses). But the logic of the process leads us to believe that the most accurate (or the most likely value) would be obtained with S0 -102 (4.982 million solar masses). And even more, we think that this value would be a lower bound of the actual mass!

Conclusion

We find that the law of universal gravitation of Newton remains today as a powerful tool for understanding our universe. The mathematics associated with it is not difficult to develop and is perfectly accessible to students in middle school.

This work is a clear example of interdisciplinarity in which the students of High School can acquire skills, discover and marvel at a time.

We attach spreadsheets about the three sections of this research: [Solar system](#), [Exoplanets](#) y [Sagittarius A*](#)

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