<u>Title:</u>

A transit of Venus quite affordable

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Abstract

Calculating the Earth-Sun distance that can be made from a transit of Venus is undoubtedly one of the greatest achievements of modern astronomy because, based on that knowledge, we can get a real idea of the size of our solar system and the size of the worlds who have joined us.

However, it requires advanced knowledge of mathematics and spherical trigonometry to perform these procedures. These are insurmountable difficulties for students of ESO and baccalaureate.

The new method presented in this research allows us to make a quite acceptable estimate of the magnitudes found by other methods that are more stringent, and we believe that this is a great exercise that can fully understand our students and helps to awaken scientific vocations and interest in geometry and calculus.

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Development

Introduction

One of the celestial stars that more has impressed the man from the first civilizations has been Venus. His showy luminosity and his presence to the dawn or to the dusk, day after day in different cycles besides the periodicity in his appearances, have done that were attributing numerous legends and histories to themselves to Venus.

It attracts our attention of very showy form the fact that some ancient civilizations, as the Maya, were predicting the exact guidelines of his appearance throughout the time. In fact, one of his calendars was based on cycles of 584 days, a calendar so exact that, in the course of 481 years, just would turn aside approximately 2 hours.

First analysis

The first aim that we us mark on having realized this work of investigation is to verify the reason for which the transits of Venus follow throughout the time such a particular dynamics and, therefore, they can turn out to be predictable.

Venus's behavior in our sky is the following one

- For several months it is, habitually, the first heavenly body that turns out to be visible after the west.
- Goes approaching, little by little, towards the Sun up to disappearing (low conjunction: happening ahead from him).
- A little time later it returns to become visible, though this time for the East, for nine months to the dawn.
- Disappears now, once again, but this time behind the Sun (top conjunction) and, after seven weeks, returns to be observable, again, to the dusk.
- East cycle lasts in total: 584 days, period about which we have spoken previously.

This regularity owes to the fact of which the Earth and Venus have orbital different periods. Venus possesses one possesses an orbital period of almost 225 days whereas the Earth is late slightly more than 365 days in completing a return about the Sun. Therefore, Venus goes to improve little by little to the Earth in his way about the Sun.
To be more exact:

Orbital astral period of VENUS $T_v = 224{,}701 days$

Orbital astral period of the EARTH $T_v = 365{,}256 days$

Thus, we know that, when the Earth has completed a return about the Sun, Venus already has given slightly more than one return and a half.
We do not propose to quarrel how much time is necessary in order that Venus "scope to the Earth "(new low conjunction).
Dividing 360 º by the orbital period of every planet we will find the central angle (with regard to the Sun) that completes every planet in one day:

$$T_v = 224{,}701 days \Rightarrow \alpha = 1{,}602129^o / day$$

$$T_v = 365{,}256 days \Rightarrow \alpha_v = 0{,}985610^o / day$$

Now we can choose to follow two different ways with which to find the days that both planets are late in returning to be:

$x = n^o$ of days

1. Using the difference between the angle crossed by Venus and the tour for the Earth:

$$(1{,}602129 - 0{,}985610)x = 360$$
$$0{,}616519x = 360$$
$$x = 583{,}92361 days$$

2. Using that Venus has given more than two returns when they return to find and the Earth only one and something more, we equalize the angle that they will have in this moment:

$$1{,}602129x - 720 = 0{,}985610x - 360$$
$$1{,}602129x - 0{,}985610x = 360$$
$$0{,}616519x = 360$$
$$x = 583{,}92361 days$$

For any of these two methods, we have just found the time that both planets are late in coinciding with his trip about the Sun.

But then: for what motive there is no Venus's traffic, approximately, every year and a half terrestrial?

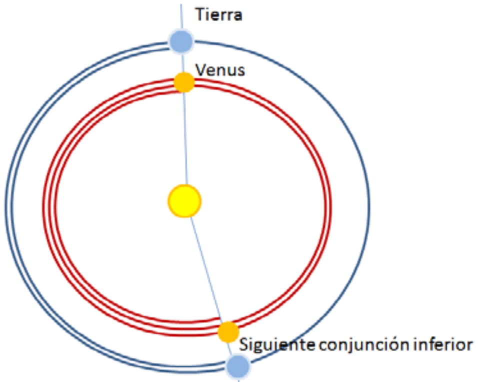


Figure 1. Times for a new low conjunction

We find the response in the fact that Venus's orbit is inclined 3,395 ° with regard to of the Earth. Venus's traffic, in consequence, only can take place in points near to the nodes of the orbit, where both planets place "to the same height ".

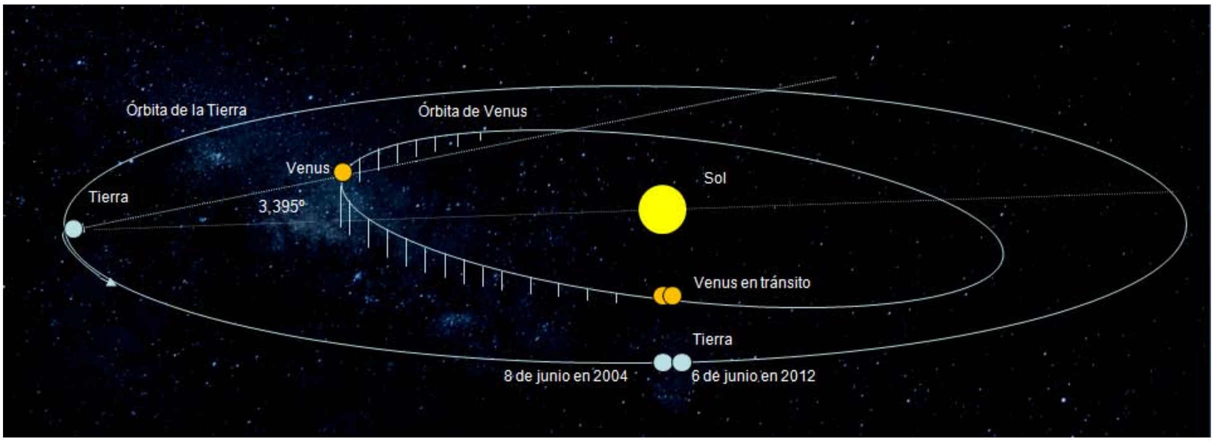


Figure 2. Relative inclination of the orbits

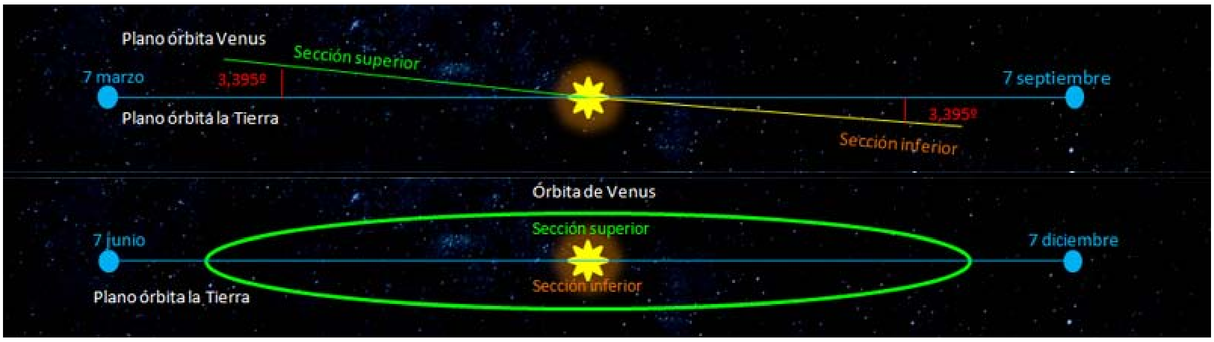


Figure 3. Relative inclination of the orbits

If we consider (since we have seen before) that the Earth and Venus are late almost 584 days in being in his way about the Sun, not in all the occasions in which they are they do it in a node or in his proximities, but they must spend five of these meetings in order that again they are in this situation:

$$583{,}92361\ días = 1{,}599\ años$$
$$583{,}92361\ días \cdot 2 = 1167{,}84722\ días = 3{,}197\ años$$
$$583{,}92361\ días \cdot 3 = 1751{,}77083\ días = 4{,}796\ años$$
$$583{,}92361\ días \cdot 4 = 2335{,}69444\ días = 6{,}395\ años$$
$$583{,}92361\ días \cdot 5 = 2919{,}61805\ días = 7{,}993\ años$$

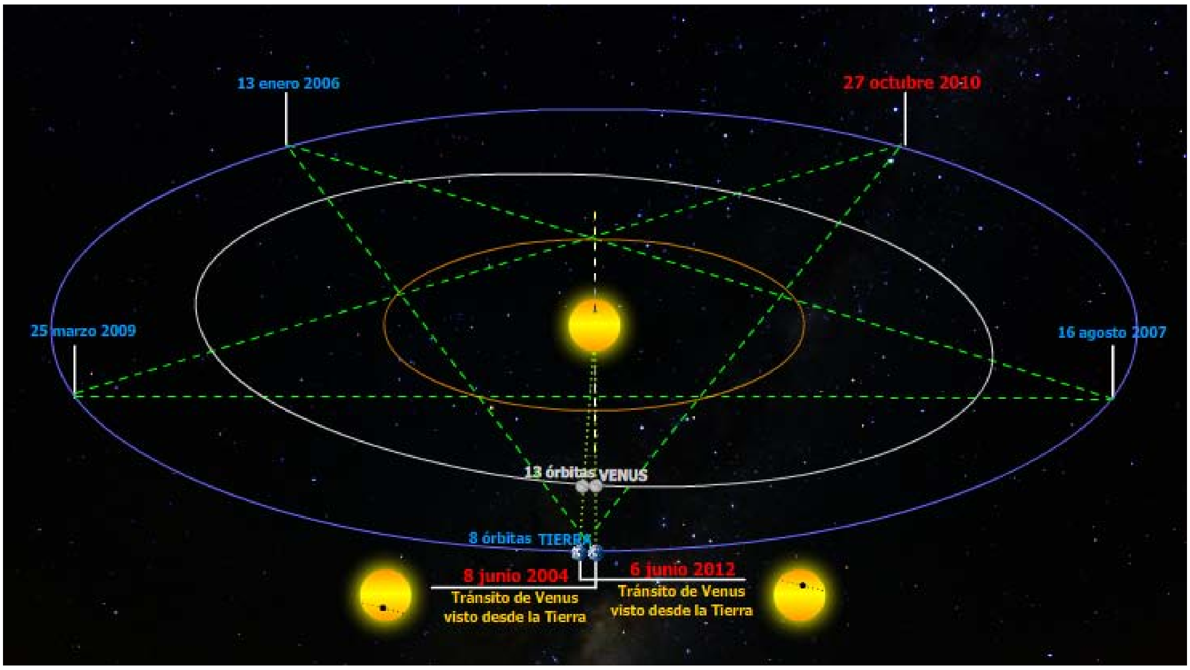


Figure 4. “Pentagonal cycle " of possible meetings

This one is the motive for the one that, after the Venus's first traffic ahead of the Sun, at the age of 8 produces the following one to himself.

Nevertheless, since we can observe, they are not 8 years exactly, but there is a small difference: 8 years suppose a period of 2922,048 days, whereas the number that we have obtained is 2919,618 days; it is to say: it has gone forward approximately 2 days and a half. This one is the reason for which, when we observe two transits of Venus consecutive, Venus does not cross the solar disc for the same place but it does it to different height.

Now then, passed other 8 more years already we are not in the same conditions in order that one returns to repeat the same spatial configuration. But regardless, Venus's new traffic will return to happen when pass slightly more than 100 years. This is like that because Venus is delaying progressively his step along the nodes. On the other hand, we must think that the following traffic will happen in the node opposite to that one in which the two took place transits previous.

If we analyze a graph similar to the previous one, the points of the new meeting come closer the node that we are referring to.

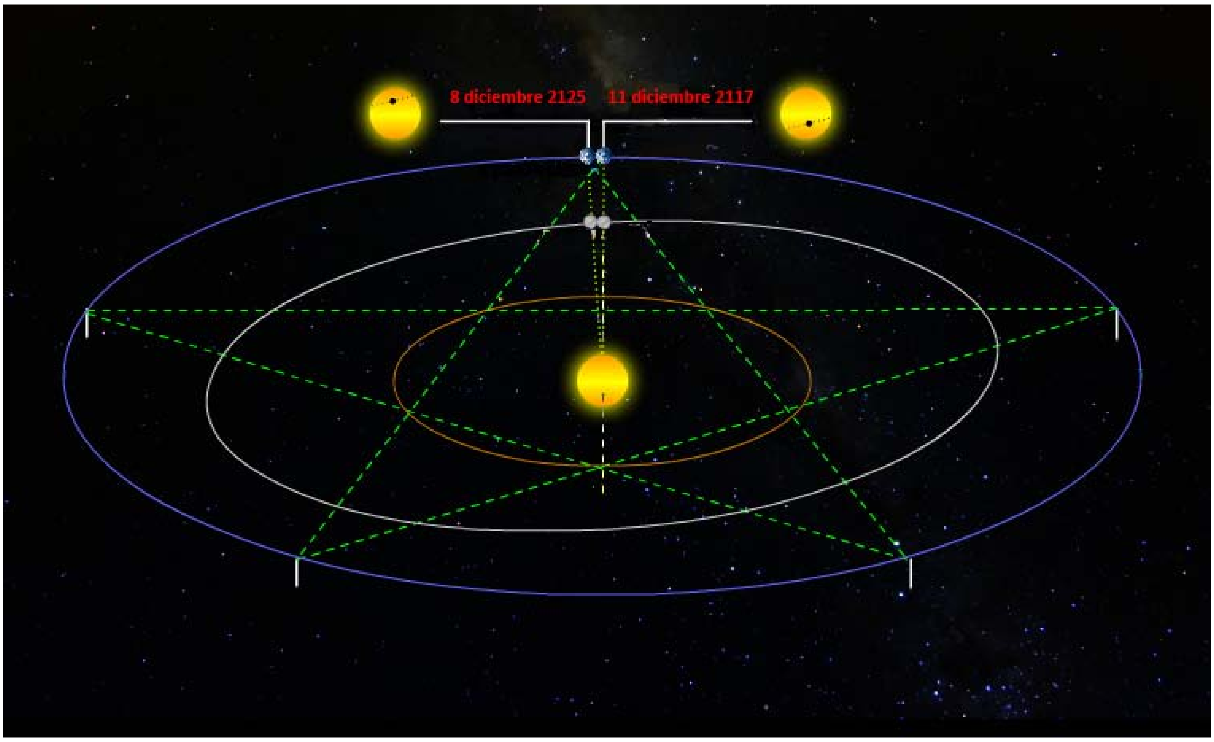


Figure 5. " Pentagonal cycle " 113 years later

Let's see which the situation for this point is in the following table:

Passed steps	Nº of passed days	Passed years	Passed steps	Nº of passed days	Passed years
1	583,92361	1,59866945	51	29780,1041	81,5321421
6	3503,54166	9,59201672	56	32699,7222	89,5254894
11	6423,15971	17,585364	61	35619,3402	97,5188367
16	9342,77776	25,5787113	66	38538,9583	105,512184
21	12262,3958	33,5720585	**71**	41458,5763	**113,505531**
26	15182,0139	41,5654058	**76**	44378,1944	**121,498878**
31	18101,6319	49,5587531	81	47297,8124	129,492226
36	21021,25	57,5521003	86	50217,4305	137,485573
41	23940,868	65,5454476	91	53137,0485	145,47892
46	26860,4861	73,5387949	96	56056,6666	153,472268

Since we can observe in her, the following transits of Venus will take place only when they pass 113 years and a half from the traffic of 2004, that is to say, when this new step places at a height of the opposite node (when the decimal part of these numbers is near 0,5), that is to say: on December 11, 2117 and December 8, 2125 (eight years later).

Second analysis

The second aim that we us have marked has been to be able to estimate with sufficient approximation some of the distances that appear in the triple system:

Sun – Earth - Venus. Using, of item, only the radius of the Earth, we will try to find the rest of the distance

If we had foreseen this study when Venus's traffic of 2004 happened, we could have contacted other centers of observation of the world or if, during the traffic of 2012 it had could turn from our coordinates, we would have carried out these measurements; but none of these cases has been possible. Nevertheless, the Internet network possesses numerous resources to have access to this information and, therefore, to be able to carry out the calculations that we propose to realize.

During Venus's traffic they are in the habit of measuring 5 different moments: the beginning, the dip completes on the solar disc, the average point of the traffic, the beginning of the exit of the solar disc and the ending of the same one.

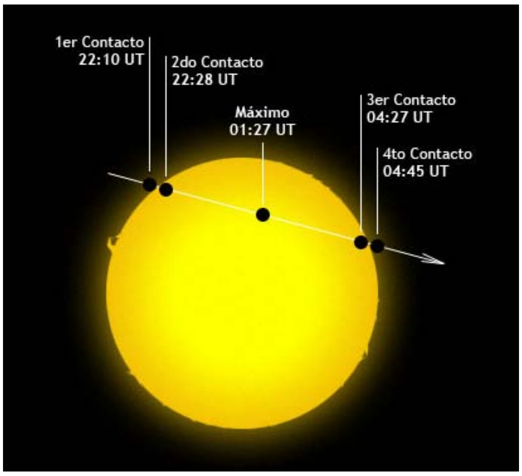


Figure 6.
Important events in the traffic

We must bear in mind that, according to which it is the point of observation from the Earth, the hours from those who produce these five events to themselves turn out to be different, and also, that the duration of Venus's traffic changes slightly between a few places and others. Therefore, we must look for the place of the most propitious planet for our observations.

We are going to take the place as an initial point of study where the solar beams fall down perpendicular on the Earth at the moment of beginning of the traffic.

For it we have resorted to two Internet pages:

http://transitofvenus.nl/wp/where-when/local-transit-times/ In this page we can see, in every point of the Earth that we choose, five moments to which we refer previously.

http://www.sunearthtools.com/dp/tools/pos_sun.php?lang=es In this another page we can obtain information it brings over of the position of the Sol in every place of the Earth and in every moment, beside seeing when there takes place the midday and the maximum height of the Sun in every instant.

Searching of meticulous form in both pages, we have found the point of the Earth where the traffic began to the midday and that in this place the Sun should present an approximate elevation of 90 º on the horizon

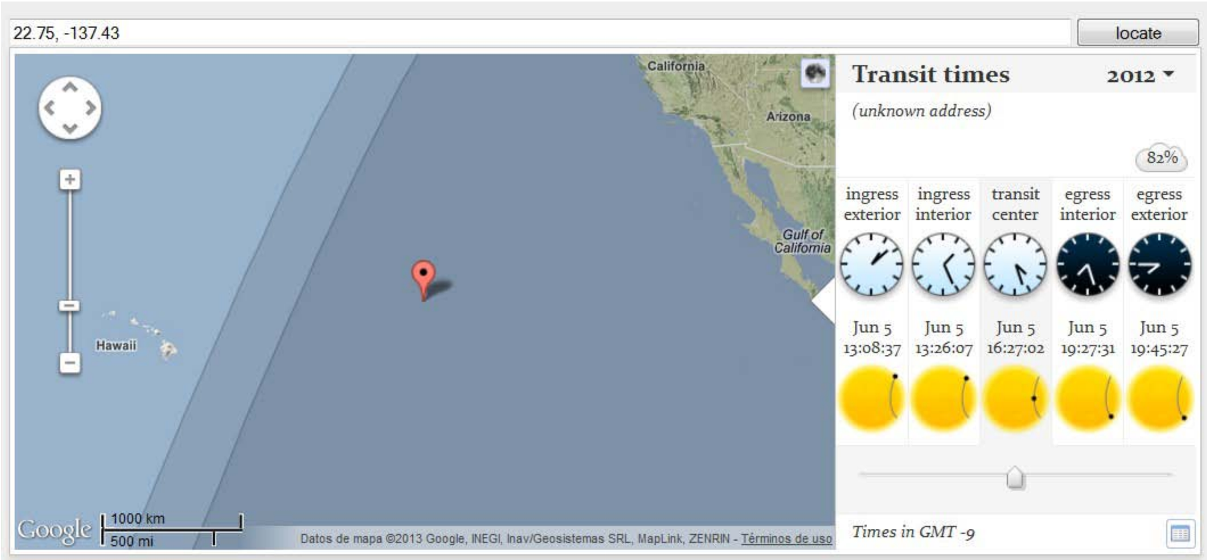


Figure 7. Initial point (A) chosen for our study

During the traffic of June 6, 2012 this point (point A) was in the Pacific Ocean, to half of way between Hawaii and California.

His coordinates are: f = 22,75º N and? To = 137,43º O. Since we can observe in the previous figure, to them 13:08:37 local time (22:08:37 hour GMT), in this point the beginning of the traffic produced to itself and also that the sunbeams fall on the terrestrial surface with an approximate inclination of 90º.

We can see also that in this place pass 17 minutes and 30 seconds between the Venus's first contact with the Sun and the full entry in the solar disc.

Our intention is to find two points (B and C) that place in same parallel (equal latitude) that they are equidistant to our first point, so that between each of these two new places with the first one there exists an approximate difference of the half of these 17 minutes and a half (approximately 8 minutes and 45 seconds) during this beginning of the traffic.

Let's observe the following graphs where both points can be observed in question, accompanied on the initial point that we have considered:

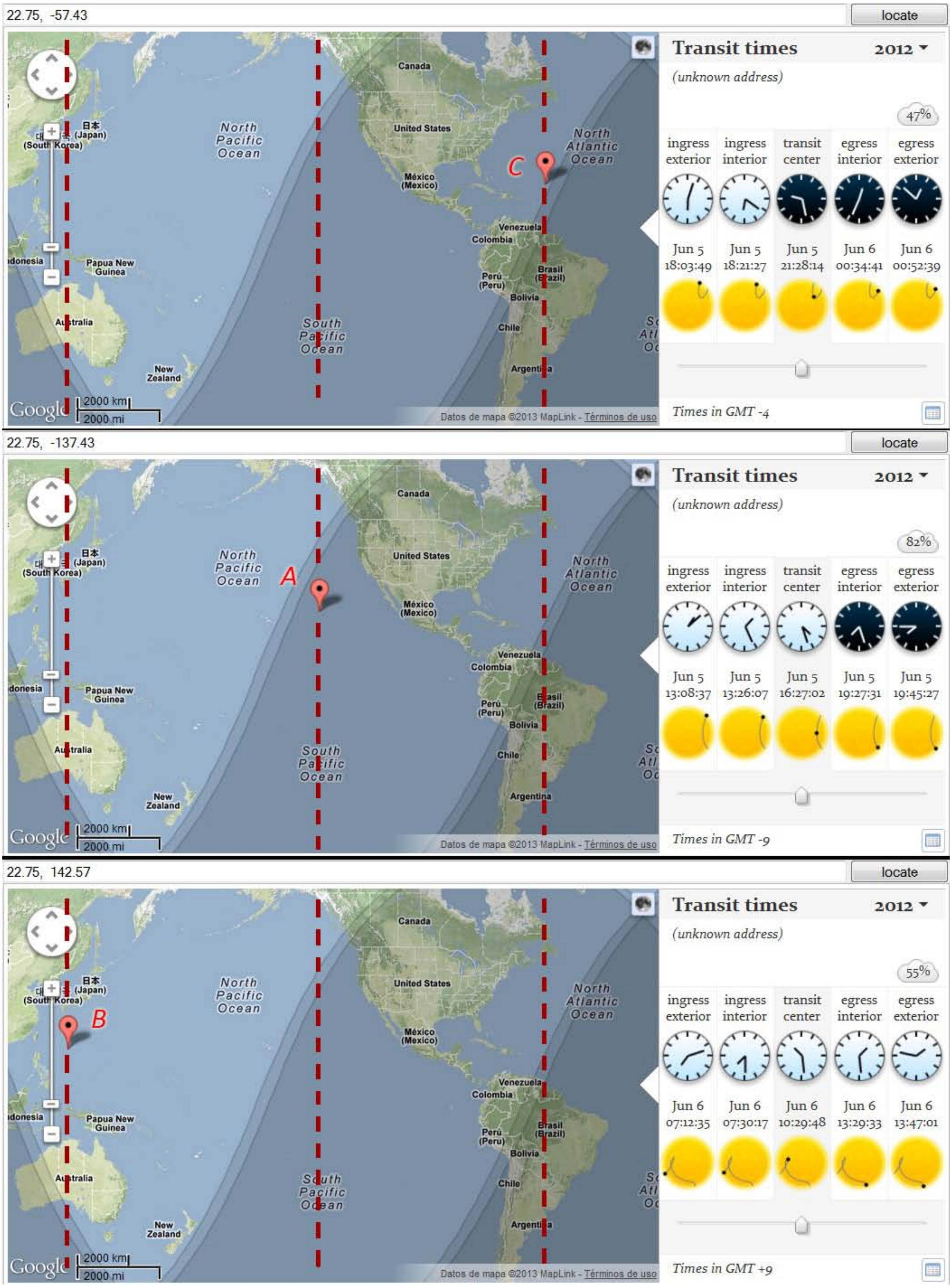


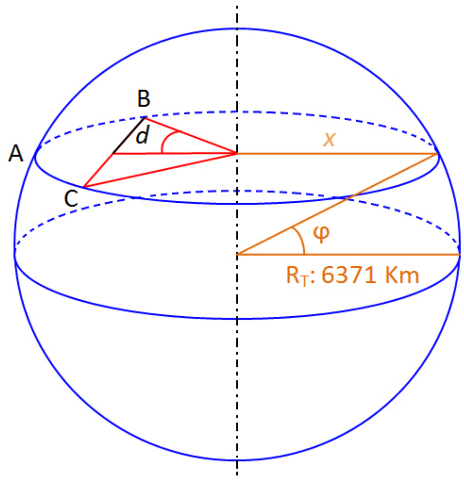
Figure 8. Points To, B and C chosen for our study

The points that we have considered to be are to a distance of 80º of the central point, that is to say: with a separation between yes of 160º.

The coordinates of these two points are: f = 22,75º N? B = 57,43º O and f = 22,75º N? C = 142,57º E, beginning the eclipse at 18:03:49 and to them 07:12:35 (local time) respectively. It is just what we cover searching: between these two points it is, actually, 8 minutes and 46 difference seconds on having begun the traffic of the eclipse.

Once chosen our places of observation, we propose to calculate the existing distance between these two points. Since both points are to the same latitude, we must verify, first, the radius of the circle that includes his parallel circle:

$$\cos \boldsymbol{\varphi} = \cos 22{,}75° = \frac{x}{6371\ km}$$
$$x = 6371 \cdot 0{,}9222$$
$$x = 5875{,}3424\ km$$



And we find the distance on line straight between B and C with the trigonometrical reason:

$$\text{sen}\ 80° = \frac{d}{5875{,}3424\ km}$$
$$d = 5875{,}3424 \cdot 0{,}9848$$
$$d = 5786{,}0827\ km$$

Figure 9.
Distances in the terrestrial globe

Therefore, the distance between B and C is of: 11572,1655 km.

We can consider now the following graph, where we estimate, in broad terms , which we try to carry out:

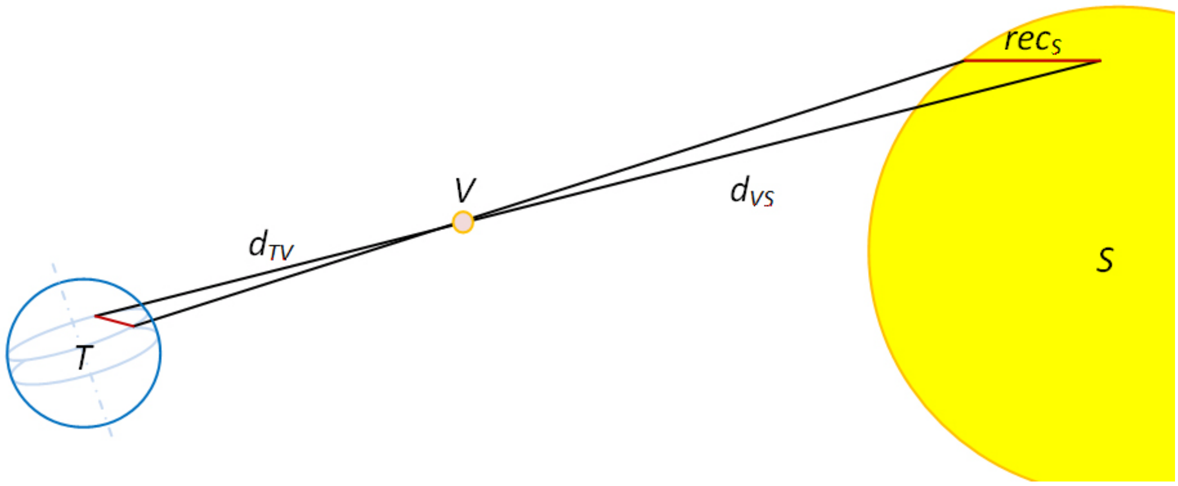


Figure 10. Basic scheme for our calculations

Following this scheme in which both triangles that we observe are in position of Such, we come to the following equality:

$$\frac{d_{TV}}{11572{,}1655} = \frac{d_{VS}}{rec_S}$$

Being

d_{TV} = Distance Earth - distance Venus
d_{vs} = Distance Venus - Sun
rec_s = Tour on the solar disc

Since we can see, we have an equation with three mysteries that to solve. For this itself we must look for the aid of other two more reasons.

The first one of them already was considered by Aristarco de Samos: the apparent diameter of the Sun observed from the Earth is of approximately 30 ' of arch. Therefore, since it shows the following graph, approximately 720 Suns would need to be able to include the whole ecliptic.

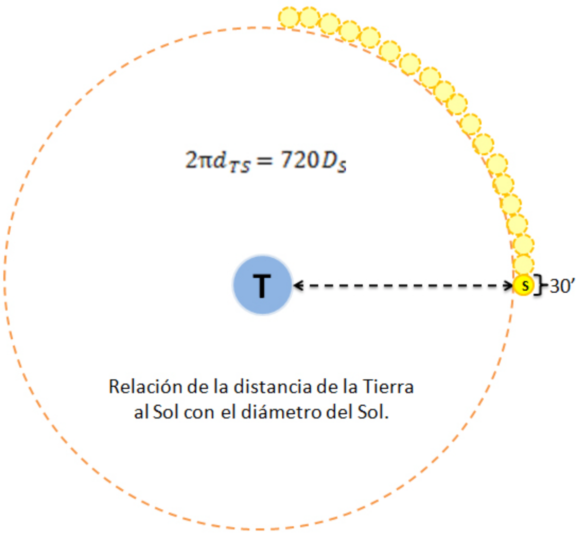


Figure 11.
720 Solar diameters

Of this form, we consider the following relation between the distance of the Earth to the Sun and the solar radius:

Length circumference = 720 Suns

$$2\pi\, d_{TS} = 720\, D_S$$

Being:

D_{TS} = Distance of The Earth - the Sun

D_s = Distance of The Sun

The second relation that we are going to apply is Kepler's third law:

$$\frac{T^2}{L^3} = \text{constant}$$

In the first pages of this work we saw that the orbital periods of the Earth and of Venus are:

T_T = 365,256 days

T_V = 224, 701 days

This way we obtain the third equality that we need:

$$\frac{365{,}256^2}{{d_{TS}}^3} = \frac{224{,}701^2}{{d_{VS}}^3} \Rightarrow \frac{133411{,}945536}{{d_{TS}}^3} = \frac{50490{,}539401}{{d_{VS}}^3}$$

$$133411{,}945536{d_{VS}}^3 = 50490{,}539401{d_{TS}}^3$$

Therefore, our three equations are:

$$\begin{cases} \dfrac{d_{TV}}{11572{,}1655} = \dfrac{d_{VS}}{rec_S} \\ 2\pi\, d_{TS} = 720\, D_S \\ 133411{,}945536{d_{VS}}^3 = 50490{,}539401{d_{TS}}^3 \end{cases}$$

But there appear expressions that, at first, we had not foreseen: and it is that we were trying to measure only a chunk of the solar disc and here us there is appearing the solar complete diameter. How to solve this problem?

The solution to this we her are going to try to find in the times that Venus's traffic is late in taking place and in the path that this one follows in his tour along the solar disc.

Noticing to http://eclipse.gsfc.nasa.gov/OH/transit12.html (information that appears in the page of the FISHNET): Venus's traffic began at 22:09:38 and finished to them 04:49:35; a total of 6 hours, 39 minutes and 57 seconds (23997 seconds).

But there is enough to us simply the time in which Venus has had his center on this disc, that is to say, approximately 17 minutes and 56 seconds less. For what refers to our calculations, the traffic would have lasted 22921 seconds.

If in addition we notice to the following graph that describes us each of these moments, we can observe to what approximate height Venus has crossed the solar disc.

Since it is possible to see, the central angle that is observed is of 106 º, therefore if we consider the half of this angle we can apply the trigonometrical reasons to put this path depending on the solar diameter.

$$\text{sen } 53° = \frac{x}{R_S}$$

$$x = 0{,}798635\ R_S$$

And the path that Venus has followed has been, actually, 0,798635 solar diameters.

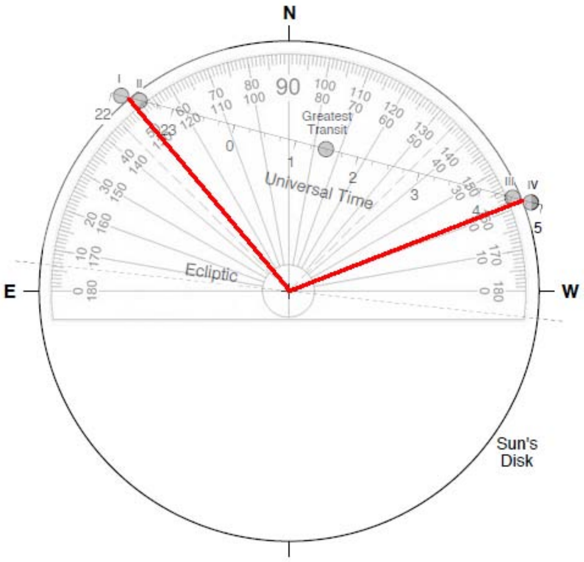


Figure 12.
Central angle of the transit

In crossing this space Venus has been 22921 second, whereas between the observations done from the point of observation B up to the C they have passed 525 seconds (8 minutes and 45 seconds). Of this form yes we will be able to work of simpler form with three previous equations, since in these 525 seconds we know that it has crossed 2,290476 % of the solar disc (it is obtained of 525/22921).

$$\begin{cases} (\mathbf{1}) \quad \dfrac{d_{TV}}{11572{,}1655} = \dfrac{d_{VS}}{rec_S} \Rightarrow 11572{,}1655 d_{VS} = d_{TV} rec_S \\ \Rightarrow 11572{,}1655 d_{VS} = d_{TV} \dfrac{2{,}290476}{100} 0{,}798635\ D_S \\ \Rightarrow 11572{,}1655 d_{VS} = d_{TV}\ 0{,}018292543\ D_S \\ (\mathbf{2}) \quad 2\pi\ d_{TS} = 720\ D_S \\ (\mathbf{3}) \quad 133411{,}945536 {d_{VS}}^3 = 50490{,}539401 {d_{TS}}^3 \Rightarrow d_{TS} = 1{,}382489\ d_{VS} \end{cases}$$

Let's happen to work with three equations.

Since just now we have four mysteries the only thing that we have to do is to express the distance of the Earth to the Sun as sum of the distances of the Earth to Venus and of Venus to the Sun when it should be necessary.

We will start by applying the method of equalization in (2) and (3)

$$\begin{cases} 2\pi\, d_{TS} = 720\, D_S \\ d_{TS} = 1{,}382489\, d_{VS} \end{cases} \Rightarrow \begin{cases} d_{TS} = \dfrac{360\, D_S}{\pi} \\ d_{TS} = 1{,}382489\, d_{VS} \end{cases} \Rightarrow \frac{360\, D_S}{\pi} = 1{,}382489\, d_{VS}$$

$$D_S = 0{,}0120645\, d_{VS} \Rightarrow d_{VS} = 82{,}887811\, D_S\ (\mathbf{4})$$

Now, we will apply that one that we have named before with regard to the distance of the Earth to the Sun in (3) to be able to substitute later in (1) and do that one of the mysteries disappear and only we still have the solar diameter:

$$d_{TV} + d_{VS} = 1{,}382489\, d_{VS} \quad \Rightarrow \quad (\mathbf{5}) \quad d_{TV} = 0{,}382489\, d_{VS}$$

$$11572{,}1655 d_{VS} = d_{TV}\ 0{,}018292543\, D_S$$

$$\Rightarrow 11572{,}1655 d_{VS} = 0{,}382489 d_{VS}\ 0{,}018292543\, D_S$$

$$\Rightarrow 11572{,}1655 = 0{,}382489 \cdot 0{,}018292543\, D_S \Rightarrow 11572{,}1655 = 0{,}0069967\, D_S$$

$$\Rightarrow D_S = 1653947{,}05\ km$$

Therefore, now only we have to substitute in (4) and (5):

$$d_{VS} = 82{,}887811\, D_S = 137092050{,}48\ km$$

$$d_{TV} = 0{,}382489\, d_{VS} = 52436201{,}30\ km$$

Obtaining, finally, a distance of the Earth to the Sun of 189.528.251,78 km.

Final considerations

We have obtained:

- $D_S = 1.653.947{,}05\ km$ (Opposite to 1.392.000 Km) (16% of mistake)
- $d_{VS} = 137.092.050{,}48\ km$ (Opposite to 108.209.000 Km) (21% of mistake)
- $d_{TV} = 52.436.201{,}30\ km$ (Opposite to ≈40.000.000 Km) (24% of mistake)
- $d_{TS} = 189.528.251{,}78\ km$ (Opposite to 149.600.000 Km) (21% of mistake)

To the first sight we would say that they are not a few very good results. It is perfectly understandable.

But let's remember that the geometry applied for the resolution of this so complex problem has been adapted for the correct comprehension only of pupils / aces of Secondary and Baccalaureate.

Actually, during 525 seconds that we have born in mind between measurements, the Earth has moved approximately 15.750 Km and "our triangles" drawn are not, in fact, in position of Such.

Since already we improve in a beginning, this singular unpublished method that we sense beforehand with this work of investigation allows us to realize an acceptable enough estimation of the same magnitudes found by other methods that are much more rigorous.
And on the other hand, we think that this one is an excellent exercise that pupils / aces can understand our / our well and that helps to wake scientific vocations and interest up for the geometry and the calculation.

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