

## Catch a Star 2016

### Title:

### The lost planet

Authors: José María Díaz Fuentes, Andrea Moral Suárez, Natalia Serrano López y Ana Sánchez Díez

Learning center: "Santo Domingo Savio" Salesian School, 23400 Úbeda (Jaén) SPAIN

E-mail: [josemaria.diazfuentes@gmail.com](mailto:josemaria.diazfuentes@gmail.com)



## Abstract

Can our young students make deep studies on Astronomy? Can they enter in the rigorous worlds of calculus and physics in an entertaining and fun way also? Yes, we are confident: curiosity born through the fiction, contrasted investigation of possible realities, conducting numerous experiments and large doses of patience lead them to truly amazing realities.

Our project was born from a first analysis of fictional histories in fictional places. It was born by reading books and watching movies in other planetary systems around several suns.

The search for reality led them to meet multiple star systems and the discovery of extrasolar planets.

It was time to introduce the Law of Universal Gravitation and understand how this is the only law that governs the movement of all celestial bodies. This point of the project is delicate because the students are - for the first time – in front to rigorous treatments of Calculus, Physics and Geometry. However, with the preparation of a specific software, that we designed for this purpose, the tasks become quite attractive - perhaps baffling activities too - but very fun.

The attractions that a planet will experience due to the presence of two suns - for instance - let our students very surprised because they see how brutal instabilities are generated in the orbit. This method can be extended to a great variety of real situations.

## Introduction

The initial idea of our research was to propose to our students the reading of texts of fiction and the viewing of films where they appeared planets orbiting in multiple stellar systems. They were to analyze the possibilities of their existence and also if, indeed, they would be able to shelter life.

In science fiction everything is possible but the reality must, first place, be governed by the strict principles of the stability of systems. Life can not develop in such an unstable system that it can not even allow the formation of planets around it or keep them, somehow, confined and in order.

Our first objective of study would be the *Universal Gravitation law* and very soon we discovered that it entailed important difficulties:

- When the law applies only to two bodies, calculations are relatively easy and accessible.
- But when intervene three or more bodies, the problems remain without analytical solutions. However, it is possible to approach his study by performing an innumerable amount of infinitesimal calculations that give us a good approximation of reality.

In order to analyze the gravitational effects that several bodies exert on each other and to see their behavior over time, we have implemented a series of computer programs compiled in an old programming language: Turbo Pascal.

This language allows us, in a formal way, to introduce our students in the programming languages and combine knowledge of Astronomy, Physics, Mathematics, Geometry and ICT.

On the other hand, we do that Turbo Pascal operates from an "aseptic" start of MS DOS getting the calculation processes to be carried out at the highest possible speed; And this requirement is fundamental since many millions of complex calculations have been carried out for our studies.

Our research project has three parts:

- I. Study of stable extrasolar planetary systems and conclusions
- II. Explanation of some peculiarities of our own solar system
- III. Annex: Construction of working tools and confirmation of their applicability

## Work development

### Stability studies with three bodies

Even in the simplest case, that of two stars orbiting around a common mass center, we can consider a large number of cases. Stars can have the same mass or be very different and the choice of the distance between them and their speeds offers us a wide range of possibilities. In the bellow explanations we will show only some notable choices that will guide us about what would happen in many other situations.

#### Studying case 1

This is the simplest case we imagine: Two identical stars orbiting around a common mass center at a predefined distance (80 million km).

$$M_{A-Sun} = M_B = 1,9891 \cdot 10^{30} \text{ Kg} \quad \left| \vec{r}_{AB} \right| = 8 \cdot 10^{10} \text{ m}$$

$$r_A = \frac{M_B \cdot r_{AB}}{M_A + M_B} = (-)4 \cdot 10^{10} \text{ m} \quad y \quad r_B = \frac{M_A \cdot r_{AB}}{M_A + M_B} = 4 \cdot 10^{10} \text{ m}$$

Both stars orbit sharing a circular path, always being in diametrically opposite points, with a period:

$$T_{orbital} = \sqrt{\frac{4\pi^2 \cdot r_{AB}^3}{G(M_A + M_B)}} = 8.725.313,4 \text{ s} = 100,987 \text{ days} \quad \text{and}$$

$$v_{orbital-A} = v_B = \frac{2\pi \cdot r_A}{P} = 28.804,399 \text{ m/s}$$

In such stellar system, any planet there will always be **two** forces of attraction:

$$\left\{ \begin{array}{l} \vec{g}_A = (-)G \frac{M_A}{r_{0A}^2} \vec{u}_A \\ \vec{g}_A (g_A \cos(\varphi_{0A} + \pi), g_A \sin(\varphi_{0A} + \pi)) \end{array} \right. \quad \left\{ \begin{array}{l} \vec{g}_B = (-)G \frac{M_B}{r_{0B}^2} \vec{u}_B \\ \vec{g}_B (g_B \cos(\varphi_{0B} + \pi), g_B \sin(\varphi_{0B} + \pi)) \end{array} \right.$$

$$\left\{ \begin{array}{l} \vec{g}_{A+B} (g_x, g_y) \\ g_x = g_A \cos(\varphi_{0A} + \pi) + g_B \cos(\varphi_{0B} + \pi) \\ g_y = g_A \sin(\varphi_{0A} + \pi) + g_B \sin(\varphi_{0B} + \pi) \end{array} \right.$$

(Position vectors are referred to each star and arguments referred to them as well and our calculation routines perform the *vector sum* of the two accelerations)

In the following figure we see how an imaginary Mars seems to be ideal when describing a circular orbit around the CDM of the double system.

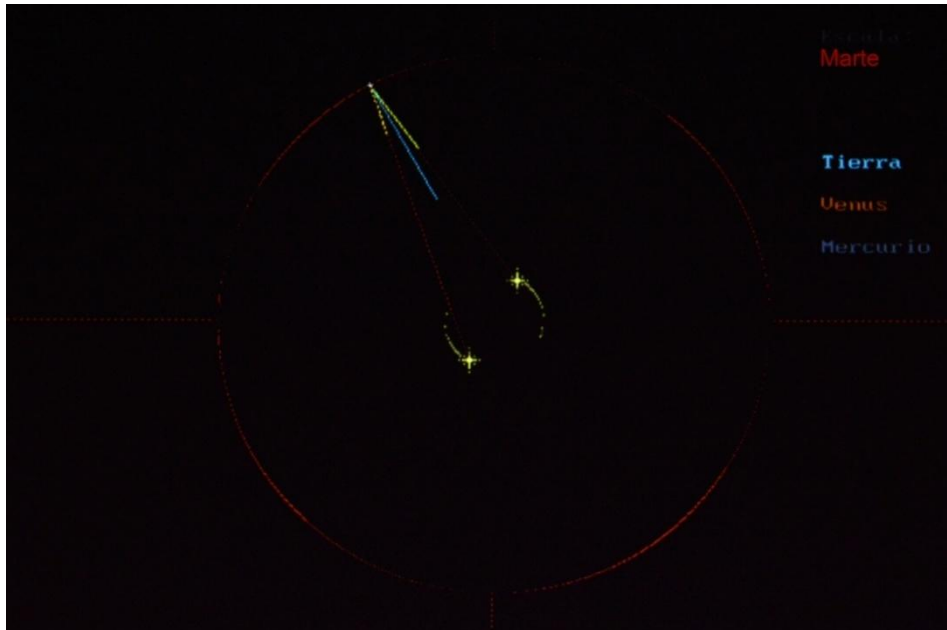


Figure 1: Sum of forces in a binary stellar system

[Video: Marte y dos soles](#) 

On the contrary, for an imaginary Venus we have been **unable** to find parameters that place it in a stable orbit. The instabilities caused by these two stars are so strong that their movement always ends up impacting with some of them or is expelled from the system.

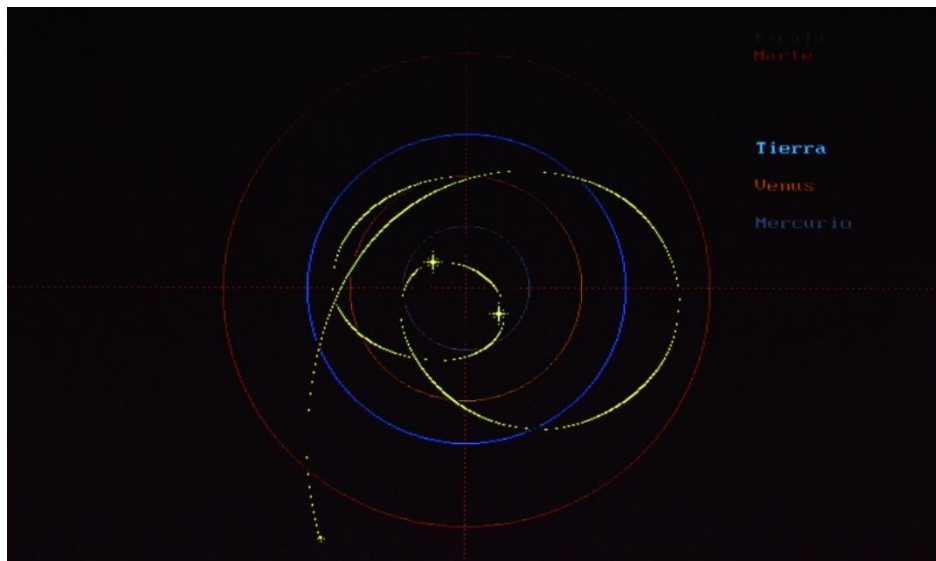
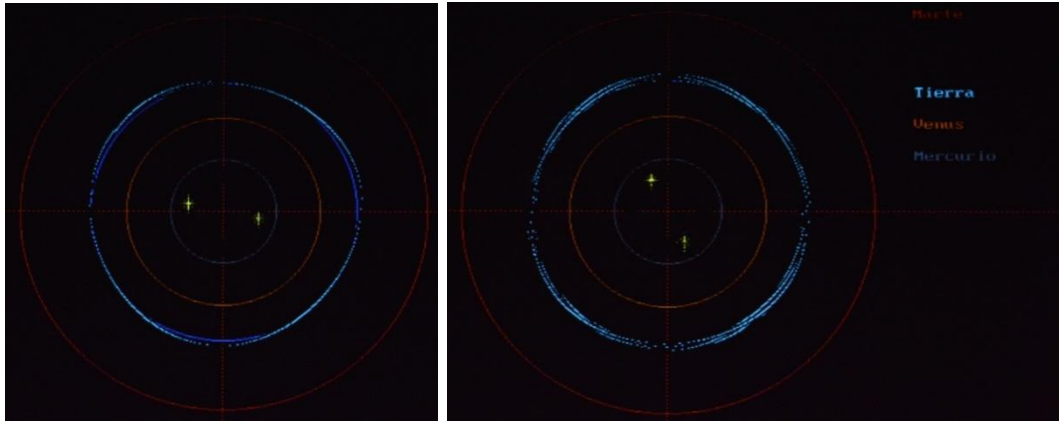


Figure 2: Expulsion of a nearby planet

[Video: Venus expulsado!](#) 

Of course with a hypothetical Mercury we do not try!

Instead, our Earth would remain in a region of *near-stability*. Although it would suffer very intense gravitational pulls, its movement would be confined in a narrow region of almost tens millions kilometers of width.



Figures 3 and 4: Unstable orbit of a planet like earth

With continuous changes of orbit of such magnitude, we find it **very difficult** to imagine how a lot of original protoplanets could form a single planet. We suspect that instead of our Earth there would be nothing but an **asteroid belt**.

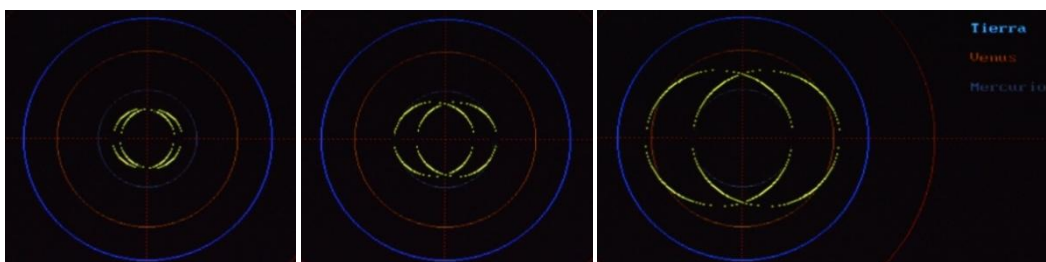
For the outer planets there would be no problems since the instabilities would hardly be noticed at great distances.

To be able to see sunsets with two identical suns in a star system like that (as in *Tatooine* in "Star Wars"), our civilization should have developed on a planet as far away (at least) as Mars (though not much further, habitability zone is always a narrow strip).

## Studying case 2

If we consider two stars even more separated ( $|\vec{r}_{AB}| > 8 \cdot 10^{10} m$ ) these problems are aggravated considerably and even Mars itself would also be unstable and could not exist.

If the velocities of the stars are different than:  $v_A = v_B \neq 28.804,399 m/s$ , these will move by describing elliptical orbits with a focus of attraction in the CDM. This situation also has bad consequences for our system of planets. We would only find stable planets in much more distant and colder regions.



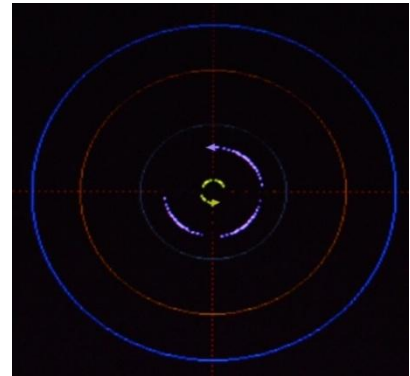
Figures 5, 6 and 7: Different possibilities in a binary stellar system

### Studying case 3

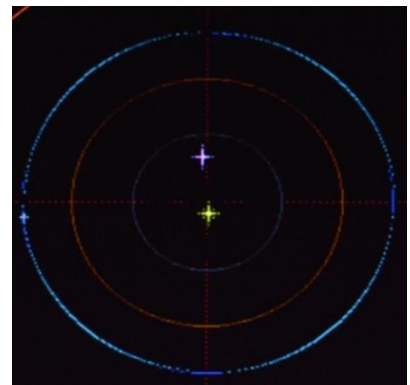
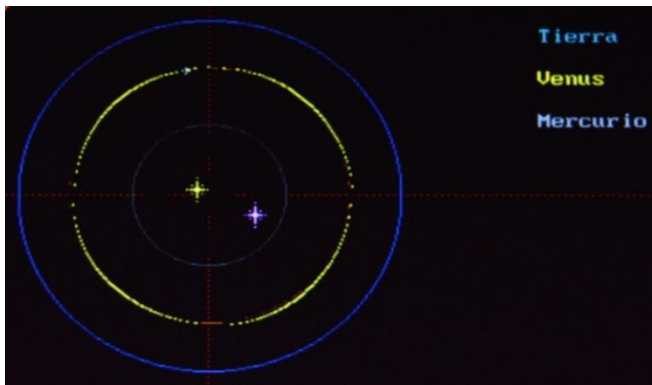
We consider a star similar to our sun accompanied by another smaller star (of mass five times smaller), orbiting around a common CDM with circular orbits. Distance between them: 80 million kilometers.

A planet like Venus would present light, but very annoying irregularities. Instead, Earth, Mars and outer planets would have "good behavior".

Video: [sistema binario I](#) 



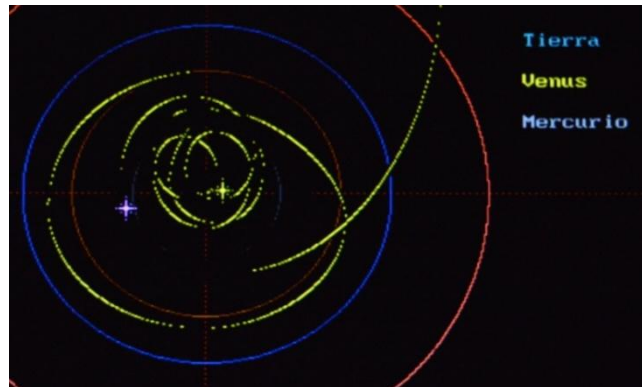
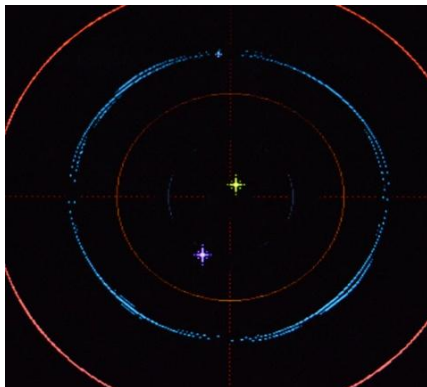
Figures 8, 9 and 10:  
Stabilities in a binary system with a small star



But with radiation coming from two stars, the habitability zone would have moved away from our surroundings. Our average temperature would be several degrees higher than the current one. Perhaps the only habitable planet (again) would be Mars.

### Studying case 4

If these two unequal stars are more separated or have elliptical orbits, these odious instabilities appear again on our inner planets. Our search for stable planets would begin, at least once, at a distance to Mars.



Figures 11 and 12: Separation of stars generates instabilities

Video: [sistema binario II](#) 



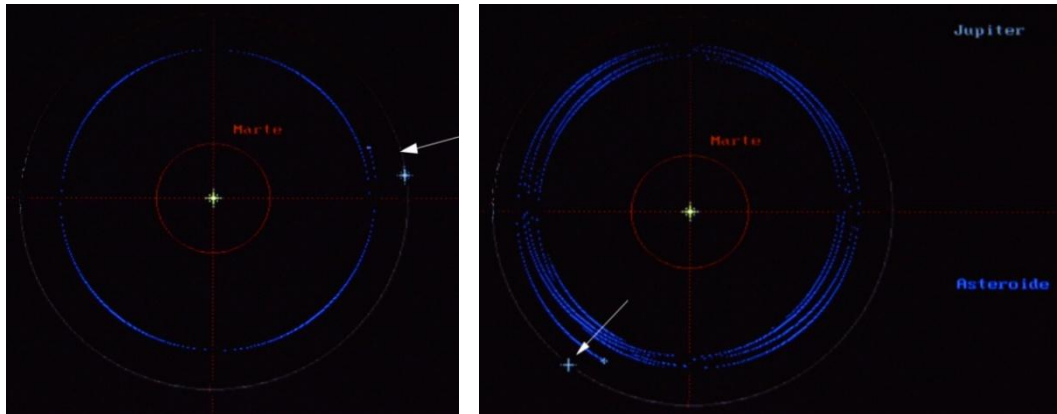
## Studying case 5

## (The lost planet)

We could see that the inner planets hardly notice their perturbations, although we know that from the purely astronomical point of view, they must be taken into account.

But the objects between Mars and Jupiter experience profound alterations.

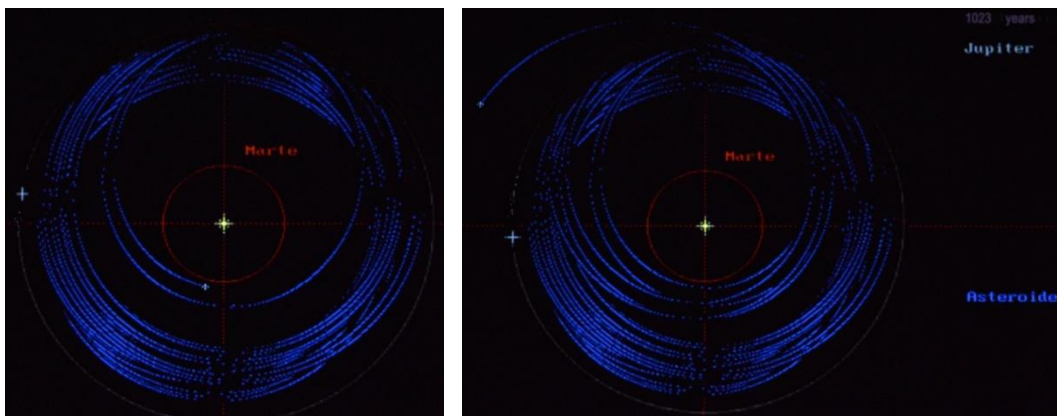
We show how an asteroid undergoes successive gravitational pulls each time it is in conjunction with Jupiter.



Figures 13 and 14: Instability of an asteroid of Jupiter's family

Video: [asteroide y Júpiter](#) 

Let's pass the time: Every time an asteroid is in conjunction with Jupiter, the interaction is very strong. There will come a day when it will end up colliding with it or diverted to the inner or outer solar system.

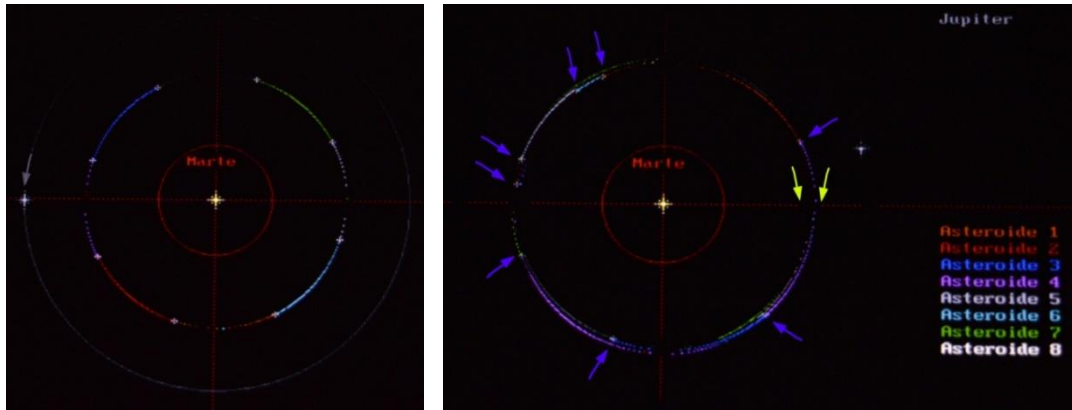


Figures 15 and 16: Instability in an asteroid after many years (1023 years)

We conclude that it is **impossible** for a planet to exist between Mars and Jupiter. And, in fact, there is not: it is our fifth planet in the solar system and is absent. If some small *protoplanets* formed one day, these ended up precipitating into the inner solar system or were expelled to distant places. This place now houses **our asteroid belt**.

We did a second test by placing eight identical asteroids to see their evolution over time.

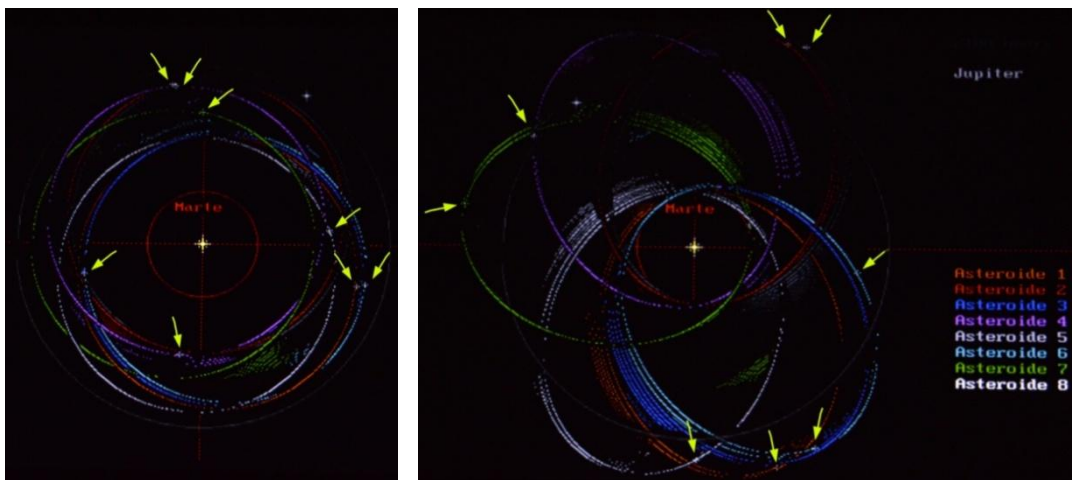
Among three hundred years, asteroids abandoned their equidistant positions away tens of millions of miles from their initial orbits. This can be seen in the following two images.



Figures 17 and 18: Simulation with eight identical asteroids

After 900 years, the orbits became strongly elliptical with supporters very close to the orbit of Jupiter.

And after 1200 years, his eccentricities increased considerably. The precession of the orbits is very visible. Observe that on one side, the orbits surpass Jupiter and, on the other, that they approach dangerously to the inner solar system. The risk of collisions is now enormous.



Figures 19 and 20: Simulation with eight identical asteroids after 1200 years

## Studying case 6

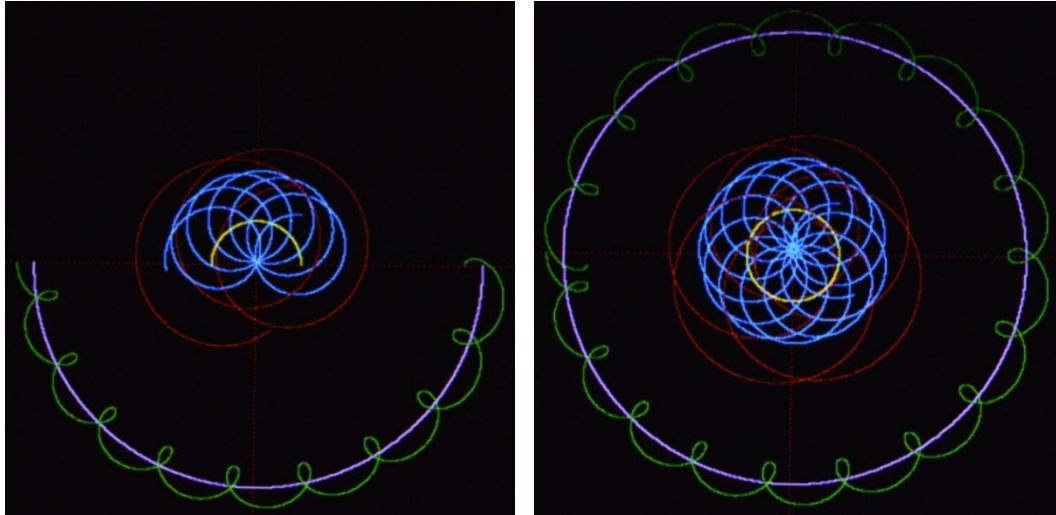
And then we continue our study of binary stars:

$$M_{A-Sun} = 1,9891 \cdot 10^{30} \text{ Kg} \quad M_B = \frac{1,9891 \cdot 10^{30}}{5} \text{ Kg}$$

$$d(A, B) = d(\text{Sun}, \text{Júpiter})$$

Both stars could be accompanied only by a **reduced** cortege of **nearby** planets.





Figures 21 and 22: A first binary system with stable subsystems

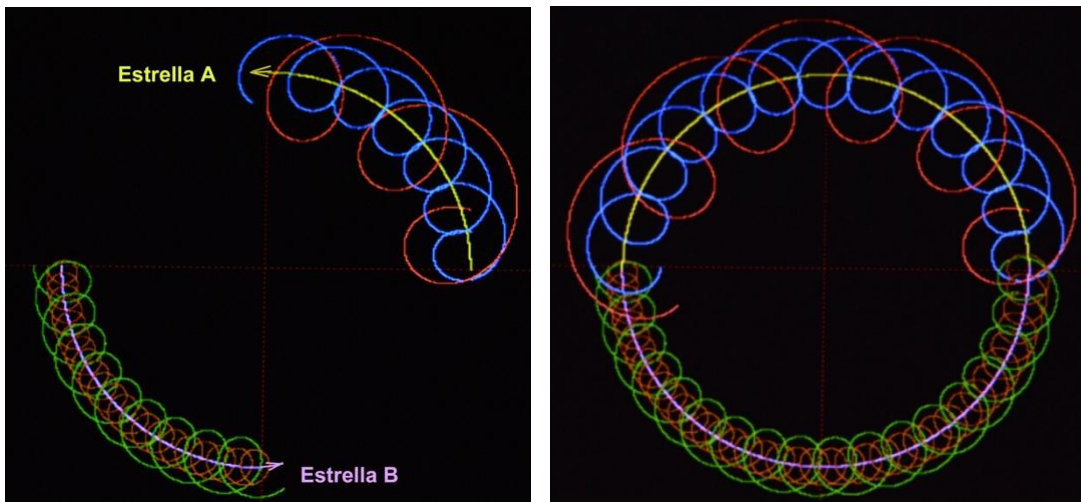
Video: [sistema binario III](#) 

Of course, if our stars are farther apart, the situation improves sensibly.

### Studying case 7

This time two identical and heavy suns orbiting at diametrically opposite points in an orbit like that of Jupiter:

$$M_{A-Sun} = M_B = 1,9891 \cdot 10^{30} \text{ Kg} \quad d(A, B) = 2 \cdot d(\text{Sun}, \text{Júpiter})$$



Figures 23 and 24: A second binary system with stable subsystems

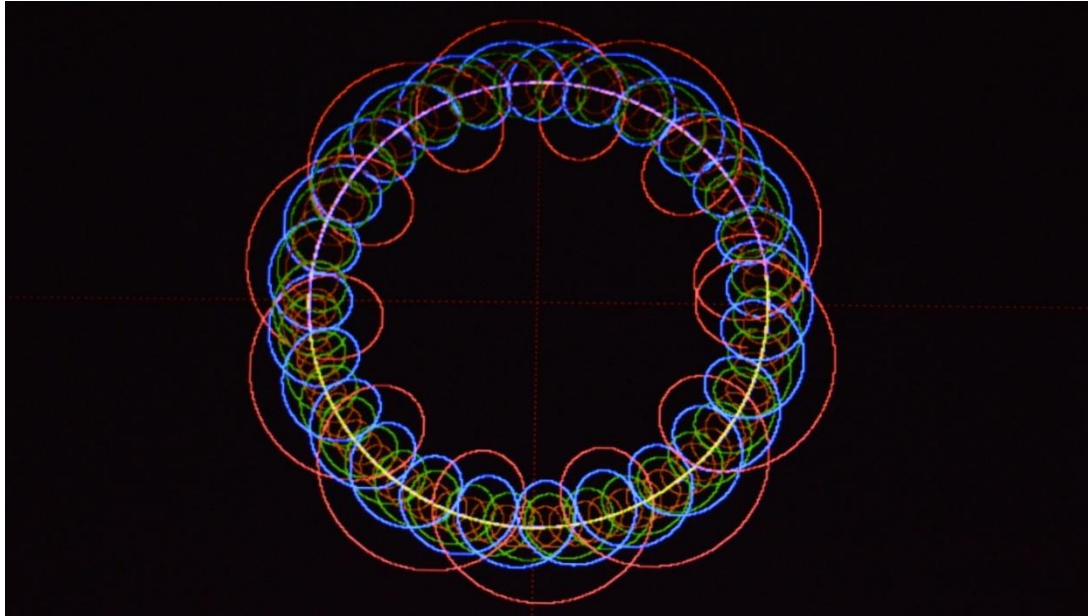


Figure 25: Completing orbits

Video: [sistema binario IV](#) 

Note, of course, that "at a greater distance, we will have greater stability".

### Summary of conclusions

We know that there are many binary stars (far apart) of long period; So, it is very likely that there are stable planetary systems around each of them.

Our research has also been extended to a greater number of bodies:

A **triple system** should consist of an exquisite balance of equidistance (highly improbable) or two nearby stars plus a third far more distant. Otherwise, the instabilities caused between them would expel one of them from the system.

And we could think of a **quadruple system** if the stars are close only in pairs and the more distant the pairs the greater the stability.

We show, finally, one example that would tell us about this latter case:

In a quadruple star system, our (blue) planet would revolve around a first pair of suns (one yellow of large mass and one orange much smaller).

At considerable distance, a pair of small stars (green and red) would orbit one around the other.

The whole system would be orbiting around a common center of mass (the origin of coordinates in our representation)

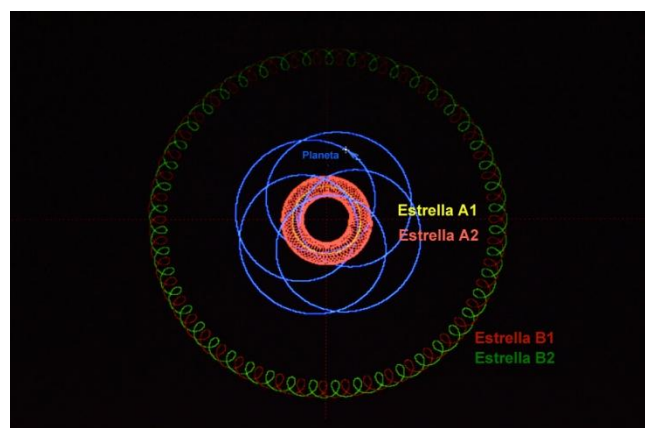
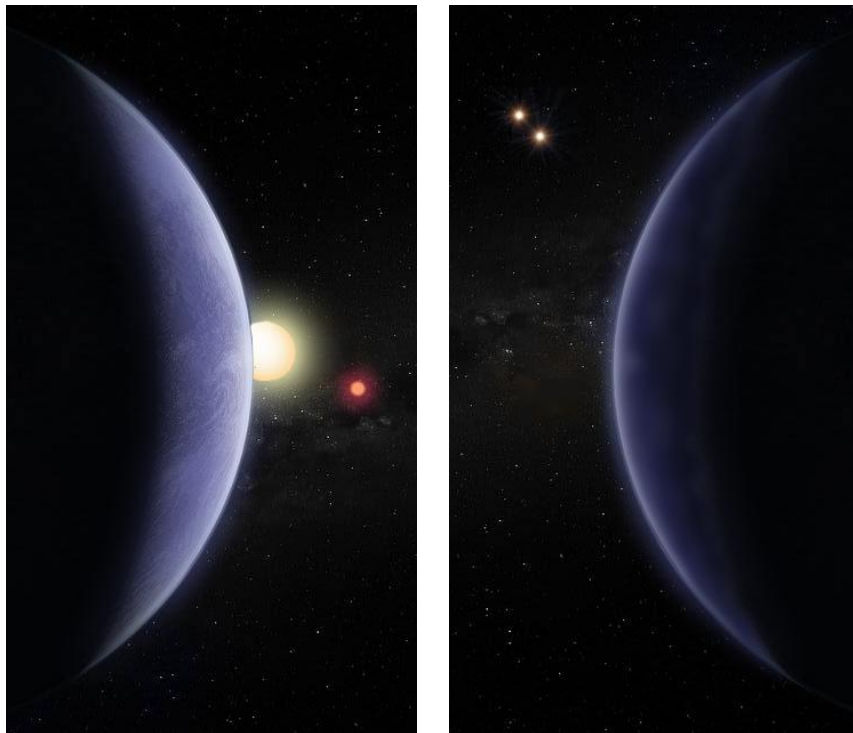


Figure 26: Quadruple star system (in pairs)

In the following two images we show how most of the sunsets and sunrises on that planet (except at times of eclipse); and also how they would be a good number of nights with two beautiful stars always in harmonic and continuous dance.



Figures 27 and 28: Imaginary quadruple star system.  
Two day suns and two distant stars

In such a stellar system, it is certain that any civilization would soon develop the precious science of astronomy.

We see that the possibilities of doing studies are almost infinite.

However, we believe that, from our humble possibilities, we have performed an analysis sufficiently representative to be able to predict - with sufficient discretion - what would happen in a large number of cases imaginable.

Finally (and this was a big surprise for all of us) we could give a reasonable explanation, within our own solar system, of why there is no "*fifth planet*" between Mars and Jupiter and yes there is, in its place, a swarm of small wandering bodies (many of them with very different orbits): **the asteroid belt**.

## References

Universal Gravitation Constant (Institute of Astrophysics Canary Islands, 2015)  
<http://www.iac.es/cosmoeduca/gravedad/fisica/fisica3.htm>

Universal Gravitation Constant (Wikipedia, november 2015)  
[https://es.wikipedia.org/wiki/Constante\\_de\\_gravitaci%C3%B3n\\_universal](https://es.wikipedia.org/wiki/Constante_de_gravitaci%C3%B3n_universal)

Universal Gravitation Constant (Report 30-june-2014, Investigación y Ciencia)  
<http://www.investigacionyciencia.es/noticias/cunto-vale-la-constante-de-la-gravitacin-universal-12207>

Orbital Parameters of Planets:

[https://es.wikipedia.org/wiki/Marte\\_%28planeta%29](https://es.wikipedia.org/wiki/Marte_%28planeta%29)

[https://es.wikipedia.org/wiki/Mercurio\\_%28planeta%29](https://es.wikipedia.org/wiki/Mercurio_%28planeta%29)

[https://es.wikipedia.org/wiki/J%C3%BApiter\\_%28planeta%29](https://es.wikipedia.org/wiki/J%C3%BApiter_%28planeta%29)

<https://es.wikipedia.org/wiki/Tierra>

[https://es.wikipedia.org/wiki/Venus\\_%28planeta%29](https://es.wikipedia.org/wiki/Venus_%28planeta%29)

Problem of the two bodies (last update 25-July-2015)  
[https://es.wikipedia.org/wiki/Problema\\_de\\_los\\_dos\\_cuerpos](https://es.wikipedia.org/wiki/Problema_de_los_dos_cuerpos)

Attached Software (Source code in PDF files)

[ORB1.PAS](#)

[ORB2.PAS](#)

[ORB3.PAS](#)

[ORB4.PAS](#)

[ORB5.PAS](#)

[ORB6.PAS](#)